

Linear Algebra; Quiz 0

1. Print your name. You must print it legibly.
2. What is your year in school and your major ?
3. What was the last math class you took ? From whom ? How did you do ?
4. What do you know about the following: matrix, vector, determinant ?
5. What other math classes do you plan to take ?
6. What grade do you honestly expect to get from this class ? Why ?

0. Print your name: _____

1. Describe geometrically the set $\{(0, 0, 1) + s(1, 0, 0) + t(0, 1, 0) \mid s, t \in R\}$.

2. Find a formula using vectors describing the line $y = 3x + 1$.

3. Describe geometrically the set of vectors $v \in R^2$ with $(1, 1) \cdot v \geq |v|$.

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1. Express the solutions to the augmented matrix $\begin{pmatrix} 1 & 2 & 3 & 0 & 8 \\ 0 & 0 & 1 & 7 & 2 \end{pmatrix}$ using a parametric equation with translation and spanning vectors.

2. Find the reduced echelon form of the matrix $\begin{pmatrix} 1 & 0 & 2 & 0 \\ 4 & 2 & 16 & 1 \\ 0 & 0 & 1 & 5 \end{pmatrix}$

3. Suppose $a \neq 0$ but $ad - bc = 0$. Find the reduced echelon form of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

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1. Express $(2, 6, 5)$ as a linear combination of $(1, 0, -2)$ and $(0, 2, 3)$. Don't guess. Explain your reasoning.

2. Find the nullspace of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

3. Suppose $A = \begin{pmatrix} 1 & -2 & 3 \\ 7 & \frac{1}{2} & 4 \\ 0 & 0 & 6 \\ 9 & 1 & 0 \end{pmatrix}$ and $X = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$. Compute the product AX .

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1. Prove that $\{(1, 0, 0, 0), (0, 2, 3, 0), (0, 4, 5, 0)\}$ is linearly independent.

2. Find a spanning set for the nullspace of the matrix $\begin{pmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

3. Let P_2 be the vector space of polynomials of degree ≤ 2 . Prove $\{1, x, x^2\}$ spans P_2 .

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1. Let A be a 7×5 -matrix. Suppose its nullspace is a line. Find the dimension of its row space. Of what Euclidean space is its row space a subspace (i.e., R^n where $n = ?$) ?

2. Suppose $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$ row reduces to $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Find a basis for the column space of the matrix A .

3. 2. Suppose $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$ row reduces to $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Find a basis for the row space of the matrix A .

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1. Find the matrix of the linear map written with variables as $f(x, y, z) = (3y - 5z, 2x + 4y)$.

2. If $A = \begin{pmatrix} 4 & 0 & 3 & 5 \\ \pi & 17 & 0 & \sqrt{3} \\ 2 & 1 & 3 & 0 \end{pmatrix}$ is the matrix of a linear map f , find f using variables.

3. Compute A^4 where $A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

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1. Find a basis for the image of the linear map with matrix $A = \begin{pmatrix} 1 & 2 & 4 & 3 \\ 2 & 4 & 9 & 7 \\ 3 & 6 & 13 & 10 \end{pmatrix}$

2. Find all matrices that commute with $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

3. Suppose A and B are $(n \times n)$ -matrices with $AB = I_{n \times n}$, the $(n \times n)$ -identity matrix. Find the nullspace of B .

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1. Find the inverse, if it exists, of $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 5 & 0 \\ 1 & 3 & 0 \end{pmatrix}$

2. Suppose A and B are invertible $(n \times n)$ -matrices. Prove that AB is invertible.

3. Suppose A is an $(n \times n)$ -matrix with $A^3 = 0_{n \times n}$. Prove that $B = I_{n \times n} - A$ is invertible with inverse $I_{n \times n} + A + A^2$.

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1. Let $\vec{v}_1 = (2, -1, -1, -1)$, $\vec{v}_2 = (1, 3, 3, -4)$, $\vec{v}_3 = (1, 1, 0, 1)$, and $\vec{v}_4 = (1, -2, 3, 1)$. Prove that $\langle \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \rangle$ is orthogonal.

2. Find the inverse of $\begin{pmatrix} 2 & 1 & 1 & 1 \\ -1 & 3 & 1 & -2 \\ -1 & 3 & 0 & 3 \\ -1 & -4 & 1 & 1 \end{pmatrix}$ using the fact from problem #1 that the columns are orthogonal.

3. Find a matrix expression such as a product of matrices (note: do **not** multiply out the expression) for a linear transformation of 4-dimensional space which has eigenvalues π , $\sqrt{2}$, -278 , and $\frac{3}{17}$ with corresponding eigenvectors \vec{v}_1 , \vec{v}_2, \vec{v}_3 , and \vec{v}_4 .