Prof. S. Brick **Math 238**
Fall '09 **section 101**

DE Exam 1 Review

Show all of your work, and explain your reasoning. In some cases an implicit solution suffices.

1. Solve $y' = \sqrt{x} \cdot y + \sqrt{x}$ where $x > 0$ and $y > 0$.

2. Draw the phase diagram of the autonomous differential equation $y' = (10 - y)(y - 3)$ and use it to sketch solution curves. (Here $x \geq 0$, but y may be negative.) Identify and classify the types of equilibrium solutions.

3. Solve $xy' - y = x$

4. Solve $y' = 2y(4 - y)$

5. Suppose a motorboat is moving at 30 fps when its motor dies. Five seconds later it is going 15 fps. Assume that the only force acting on the boat (resistance) is proportional to velocity. How far will the boat coast ?

6. Solve $\left(\sqrt{y} + \frac{x}{y}\right)y' = -12 - \ln(y)$.

7. Solve $xy' + 6y = 3xy^{\frac{4}{3}}$

8. Solve $x^2y' = x^2 + y^2 + xy$.

9. Verify that the function $y = (x + C)\cos(x)$, where C is a constant, is a solution to $y' + (y \cdot \tan(x)) = \cos(x)$. Then find the value of C such that the initial condition $y(\pi) = -3\pi$ is satisfied.

10. Solve $x^2y' = 1 + y^2$

11. A tank initially contains 100 gallons of pure water. Brine containing $\frac{1}{5}$ pounds of salt per gallon enters the tank at the rate of 2 gallons per minute. A perfectly mixed solution leaves the tank at the rate of 3 gallons per minute. Let $s(t)$ be the amount of salt in the tank after t minutes. Set up a differential equation with initial conditions (i.e., an IVP) for s for $0 \leq t \leq 100$. Solve the resulting IVP.

12. Verify that the function $y = Ce^{-x} + x - 1$, where C is a constant, is a solution to $y' = x - y$. Find C such that the initial condition $y(0) = 4$ is satisfied.

13. Solve $x^2y' + 2xy = 5y^4$

14. Where does the slope field for $\dfrac{dy}{dx} = \dfrac{x^2y + y^2x}{3x + y}$ have horizontal segments or directions ?

15. Solve $x^3 + (y/x) + (y^2 + \ln(x))y' = 0$.

16. Draw the phase diagram of the autonomous differential equation $y' = (y-7)^2(3-y)(5+y)$ and use it to sketch solution curves. (Here $x \geq 0$, but y may be negative.) Identify and classify the types of equilibrium solutions.

17. Solve $y' = x^2 + y$

18. A stone is dropped at an initial height of h feet. Use differential equations to show that it hits the ground with a speed of $\sqrt{2gh}$.

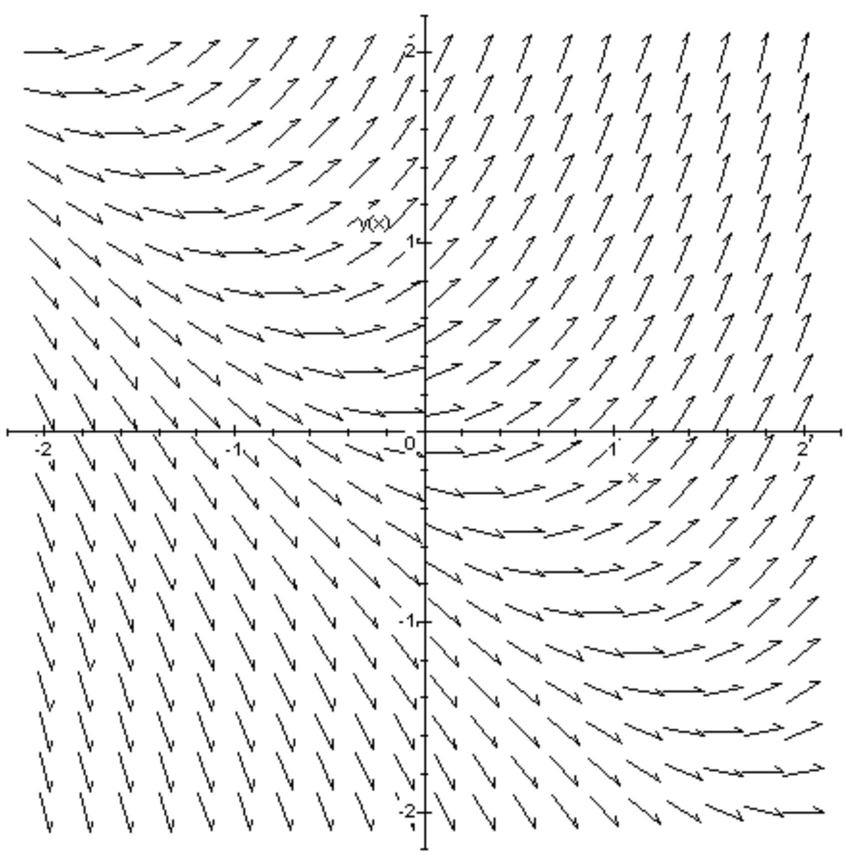
19. Solve $xy' + (2x - 3)y = x^4$ for $x > 0$

20. Solve $(x - y)y' = x + y$

21. A car traveling 88 ft/sec skids 176 feet after its brakes are applied. Set up differential equations that model the situation under the assumption that the deceleration is constant. Then use the equations to find out the amount of time that the car skids.

22. Sketch a solution to the IVP $y' = x + y$ with $y(-1) = \frac{1}{2}$ on the slope field given below. Be sure to show the initial condition.

Slope field for dy/dx = x + y



23. Solve $xy'' = y'$

24. Solve $\dfrac{dy}{dx} = -\dfrac{e^x + ye^{xy}}{e^y + xe^{xy}}$

25. Solve $y' + y\cot(x)y = \cos(x)$

26. Solve $y' + y = \cos(x)$

27. Solve $\quad \cos(y) \cdot y' = \dfrac{4x^2 + \sin^2(y)}{2x\sin(y)}$

28. Solve $y' + 2xy = 2x$ with $y(0) = -2$

29. Solve $xy' + 6y = 3xy^{4/3}$.

30. Solve $y' + \dfrac{4y}{x} = x + 3$.

31. Review all the homework, old quizzes, online exams, and everything else.