

1. Show that $y_1 = x^2$ and $y_2 = x^{-3}$ are linearly independent.
2. Solve the initial value problem $x^2y'' + 2xy' - 6y = 0$ with $y(1) = 10$ and $y'(1) = 15$, given that $y_1 = x^2$ and $y_2 = x^{-3}$ are solutions (which you should check).
3. Solve $y'' - y' - 12y = 0$.
4. Solve $y'' - 2y' + y = 2e^x$
5. Find a non-trivial linear homogeneous ODE with constant coefficients for which $y = e^{2x} \sin(3x)$ is a solution.
6. Consider the equation $y' = x^2 - y^2$ with initial conditions $y(0) = .2$. Use Euler's method with step size .25 to approximate $y(1)$.
7. A boat is moving 40 feet per second when its motor quits. Ten seconds later the boat has slowed to 20 feet per second. Assume that the resistance the boat encounters while coasting is proportional to its velocity. Set up and solve a differential equation for the velocity of the boat. How far does it coast ?
8. Find a non-trivial homogeneous second order linear differential equation that has x^2 and x^{-2} as solutions.
9. Solve $y'' - 3y' + 2y = \sin(2x)$.
10. Solve $y'' - 9y = 4e^{3x}$.
11. True or False:
 - (a). The characteristic equation of the ODE corresponding to the free undamped motion (i.e., simple harmonic motion) of a mass on a spring has two real roots.
 - (b). A second order homogeneous linear ODE with constant coefficients always has two linearly independent solutions.
 - (c). Suppose the characteristic equation of a homogeneous linear ODE with constant

coefficients has only real roots, all of which are negative. Then any solution $y = y_1(x)$ has the property that $\lim_{x \rightarrow +\infty} y_1(x) = 0$

12. Find a non-trivial linear homogeneous ODE with constant coefficients for which $y = \sqrt{2} + 2^x$ is a solution.

13. Suppose $L[y] = x^9$ is a normalized linear second order ODE with $y_1 = x^3$ and $y_2 = x^4$ solutions to the homogeneous ODE $L[y] = 0$. Solve the ODE $L[y] = x^9$.

14. Verify that $y_1 = 1/x^2$ and $y_2 = \ln(x)/x^2$ are solutions to $x^2y'' + 5xy' + 4y = 0$. Determine whether or not y_1 and y_2 are independent.

15. Solve $y''' + 2y'' - y' - 2y = 0$

16. Solve $y'' + 4y = \sin^2(x)$ using variation of parameters.

17. Solve $3y'' + 5y' + 10y = 0$.

18. Use the method of variation of parameters to solve $y'' + 4y = \cos(3x)$.

19. Solve $y^{(iv)} - 5y'' + 4y = 0$

20. Find a non-trivial linear homogeneous ODE with constant coefficients for which $y = (e^x + 1)^2$ is a solution.

21. Solve $2y'' - 3y' + 7y = 3x^3 + 2x$

22. Find a non-trivial homogeneous second order linear differential equation that has x^3 and x^{-5} as solutions.

23. Solve $25y'' + 10y' + y = 0$

24. Review everything else.