

0. Review all the homework and quizzes.

1. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$. Find the set $A \times B$.

2. Find the cardinality of the set $\{5, \{5\}, \{5, 5\}\}$

3. Find the truth table for $(p \rightarrow (\neg q)) \rightarrow (p \vee q)$

4. Find the power set of $\{1, 2\} \cup \{2, 3\}$

5. Let A be the set of binary strings of length three that have exactly one occurrence of a one. Find the complement of A inside of the universal set $U =$ the set of all binary strings of length three.

6. Let $A_n = \{x \in Z \mid n < -x \text{ and } x > 3\}$ and $B_n = \{x \in Z \mid n < x \text{ and } x \leq 3\}$ (where Z is the set of integers). Find $A_{-5} \times B_1$ by enumerating its elements.

7. Let A be the set of all prime numbers ≤ 10 . Let B be the set of all even numbers. Taking the universal set to be the set of all integers, find $A \cap B^c$.

8. Give a partition of the power set of $\{0, 1, 2\}$ into four blocks

9. Suppose you have a set A of binary strings, none of them being the empty string. Further assume that the cardinality of A is ≥ 5 . Prove that there are at least two strings in A which start with the same symbol and also which end with the same symbol.

10. Find the truth table for $(p \wedge q) \rightarrow (\neg r)$

11. Is the number 827351862 divisible by 7. Use the divisibility test from class and show your work.

12. Let $P(x)$ be the statement “ x is taking Math 267” and let $Q(x)$ be the statement “ x is ecstatic”. If the domain of discourse is all students at South Alabama, translate into English the statement: $\forall x (P(x) \rightarrow Q(x))$ (you may treat the domain of discourse as implicit).

13. Prove by induction the following If a big chocolate bar can be broken up into N small rectangles, then any way of doing it always involves $N - 1$ breaks.

14. Let A be the set of all positive integers ≤ 25 . Let B be the set of all prime numbers. Taking the universal set to be the set of all integers, find $A \cap \overline{B}$.

15. If twelve eggs are dyed a solid color and there five colors available, what can you conclude about the number of eggs of the same color ? Give the best answer possible. What principle do you use ?

16. Let the domain of discourse be the set of all integers. Determine the truth value of the statement $(\forall x (x^2 > x)) \rightarrow (1 = 2)$

17. State the contrapositive of “If you water the lawn then it rains.”
18. By considering cases, prove that $\chi_{A \cap B}(x) = \chi_A(x) \cdot \chi_B(x)$, where χ_S denotes the characteristic function of the set S (so it takes on the value of 1 for elements in S and the value of 0 otherwise).
19. Define $f(n)$ by $f(1) = 4$ and $f(n + 1) = 3f(n) + 2$. Find $f(4)$.
20. Why should people in computer science learn about proofs by induction ? Be concise. You need not explain or elaborate. (Hint- the answer is not “to pass this course” or “to think mathematically” or “to solve problems”.)
21. Define a set S of binary strings by $1 \in S$ and $0 \in S$. Also suppose that if $w \in S$ then $0w0 \in S$ and $1w1 \in S$. Describe the resulting set S .
22. Find the power set of $\{3, \{3\}\}$
23. For what values of the digit a is the number $6523472a981521$ divisible by 3
24. Prove or disprove: the product of two odd integers is itself odd.
25. Use the Euclidean algorithm to find the gcd of 864 and 1392, showing all of your steps.

26. Find the g.c.d. and l.c.m. of a and b where $a = 3^3 \cdot 5^2 \cdot 7^4 \cdot 10^3$ and $b = 2^4 \cdot 5^2 \cdot 7$
27. Prove that $n > 1$ is not prime then it has a factor that is $\leq \sqrt{n}$
28. If the gcd of x and y is equal to 12 then can you find integers a and b such that $ax + by = 10$?
29. Describe the divisibility test for divisible by 8 and explain why it works..
30. Prove that the sum of the first n positive odd integers is equal to n^2
31. Prove that a quotient of integers must have a repeating or terminating decimal expansion. What principle do you use ?
32. Find integers x and y so that $\gcd(297, 954) = 297x + 954y$
33. Describe the recursive method that will find all primes ≤ 1000 . What works as the exit condition ?
34. Review everything else.