MA125 Calculus I Review

The final will be comprehensive but will emphasize material from Chapter 5 (~40%). This review provides examples of a few problems from Chapter 5. The solutions will be posted on Friday. While preparing for the other chapters, review previous exams, quizzes and home work.

Chapter 5

5.0 “Physics-Based Calculus”

1. **Derivatives**: Given the position function, find the velocity and acceleration function.

   An object is dropped from the top of a 100-m-high tower. Its height above ground after \( t \) sec is \( 100 - 4.9t^2 \) m. Find the velocity and acceleration of the object as a function of \( t \).

2. **Anti-Derivatives**: Given the acceleration function, find the velocity function. Given the velocity function, find the position function.

   A particle moves on a coordinate line with acceleration \( a = \frac{d^2s}{dt^2} = 15\sqrt{t} - \left(\frac{3}{\sqrt{t}}\right) \), subject to the conditions that \( \frac{ds}{dt} = 4 \) and \( s = 0 \) when \( t = 1 \). Find the velocity \( v = ds/dt \) in terms of \( t \) and the position \( s \) in terms of \( t \).

5.1 Area Approximations

3. Approximate the area under the graph of \( f(x) = \cos^2(x) \) over the interval \([-\pi/2, \pi/2]\) using a lower sum (upper sum, left endpoints, right endpoints) with four rectangles of equal width. (5.1 #1,3,7)
5.2 Sigma Notation and Limits of Finite Sums

4. Calculate the following sum: \(\sum_{k=2}^{4} \frac{\cos(k\pi)}{k-1}\). (5.2 #1,3,5, Quiz 13)

5. Express the following sum in sigma notation: \(\frac{1}{3} - \frac{1}{6} + \frac{1}{9} - \frac{1}{12} - \frac{1}{15}\). (5.2 #15, Quiz 13)

6. Find a formula for the Riemann Sum \(R_n\) for \(f(x) = x + x^2\) on the interval \([0,1]\) by dividing the interval into \(n\) equal sub-intervals and using the right-hand endpoint for each rectangle height. Then compute the area under the curve by taking the limit of these sums as \(n \to \infty\). (5.2 #39,41,43)

5.3 Computing Definite Integrals Using Geometry, and Properties of Definite Integrals

7. Compute \(\int_{-5}^{5} (\sqrt{25-x^2}) dx\) using geometry. (5.3 #15,17,19,41, Quiz 14)

8. Given that \(\int_{-1}^{1} h(x) dx = 0\) and \(\int_{-1}^{3} h(x) dx = 6\), (5.3 #9,11,13, Quiz 14)
   a. Find \(\int_{1}^{3} h(r) dr\):
   b. Find \(\int_{1}^{-1} h(s) ds\):
   c. Find \(\int_{3}^{1} h(t) dt\):

5.4 Fundamental Theorem of Calculus

9. Evaluate \(\int_{-2}^{0} (3x - 2e^{5x}) dx\) using the fundamental theorem of calculus. (i.e., evaluate the integral using the anti-derivative). (5.4 #1,3,5,17,19,29)

10. Evaluate \(\int_{0}^{\pi} 1 + \cos t dt\) using the fundamental theorem of calculus. (i.e., evaluate the integral using the anti-derivative). (5.4 #7,21,23)

11. Calculate the derivative of \(\int_{100}^{t} \cos 5x\ dx\) using the fundamental theorem of calculus. (5.4 #39,41,43,45,49,53)

12. Calculate \(\frac{d}{dx} \int_{x^3}^{0} \sin^2 t\ dt\). (5.4 #39,41,43,45,49,53)

13. Calculate \(\int_{-4}^{4} 3 + |x| dx\) using any method. (5.4 #27,57,61)

14. Find the total area between \(y = -x^2 - 2x\) and the x-axis, between \(-3 \leq x \leq 2\). (5.4 #27,57,61)