Quiz 7: 6.4 (Surface Area) and 6.5 (Work)

1. Find the area of the surface generated by revolving the curve $y = \frac{x^3}{8}$, $0 \leq y \leq 1$, about the y-axis. (5 points)

$$S = \int_0^1 2\pi \left( \frac{y^3}{8} \right) \sqrt{1 + \left( \frac{3y^2}{8} \right)^2} \, dy$$

$$= \int_0^1 \frac{\pi y^3}{4} \sqrt{1 + \frac{9y^4}{64}} \, dy$$

$$= \int_0^1 \frac{\pi y^3}{8.4} \sqrt{64 + 9y^4} \, dy = \int_{u(1)}^{u(3)} \frac{u^{3/2}}{36} \sqrt{u} \, du$$

$$u = 64 + 9y^4$$

$$du = 36y^3 \, dy$$

$$dy = \frac{du}{36y^3}$$

$$= \frac{\pi}{8.4} \left( \frac{2y^3}{3} \right) \left[ \frac{1}{3} \left( \sqrt{u} \right)^{1/2} \right]_{64}^{73}$$

$$= \frac{\pi}{2.37} \left( \frac{32}{3} \right)$$

$$= \frac{\pi}{2.37} \left( \frac{5.2}{1728} \right)$$

2. A force of 10 N will stretch a rubber band 4 cm. Assuming that Hooke's Law applies, how far will a 20-N force stretch the rubber band? How much work does it take to stretch the rubber band this far? (5 points)

10 N stretches band 4 cm:

$$F = kd$$

$$10 = k \cdot 4 \Rightarrow k = \frac{10}{4} = \frac{5}{2} \text{ N/cm}$$

How far will a 20 N force stretch the band?

$$F = kd$$

$$20 = \frac{5}{2} \cdot d$$

$$d = \frac{40}{5} = 8 \text{ cm}$$

Work done?

$$W = \int k x \, dx = \int_0^8 \frac{5x^2}{4} \, dx = \frac{5 \cdot 2^6}{2^2} > 5.2^4$$

$$W = 80 \text{ Nm}$$

$$-9 \text{ Nkm} = -85$$