1. A **topological space** consists of a set $X$ and a subset $T \subset \mathcal{P}(X)$ of the power set $\mathcal{P}(X) = 2^X$ — the set of all subsets of $X$. The elements of $T$ are called **open sets**, and they satisfy these three properties:

- $\emptyset, X \in T$,
- if $U_\alpha \in T$, $\forall \alpha \in I$, then $\bigcup_{\alpha \in I} U_\alpha \in T$.
- if $U_1, U_2, \ldots, U_n \in T$, then $\bigcap_{j=1}^n U_j \in T$.

In other words, $T$ is closed under arbitrary union and finite intersection; it contains $X$ and it contains the empty set.

2. A **Hausdorff space** is a topological space $(X, T)$ such that for any two points $x, y \in X$ with $x \neq y$ there exist open sets $U(x)$ and $V(y)$ such that $x \in U(x)$, and $y \in V(y)$, while $U(x) \cap V(y) = \emptyset$.

3. A **compact space** is a space for which every open cover has a finite subcover. So if $U_\alpha \in T$ for all $\alpha \in I$, and $X \subset \bigcup_{\alpha \in I} U_\alpha$, then there exist $\alpha_1, \alpha_2, \ldots, \alpha_n$ such that $X \subset U_1 \cup U_2 \cup \cdots \cup U_n$.

4. A **metric space** is a topological space for which the open balls form a basis for the topology. More specifically, there is a distance function $d : X \times X \hookrightarrow \mathbb{R}$, that satisfies:

- $0 \leq d(x, y)$ for all $x, y \in X$. Furthermore, $0 = d(x, y)$ if and only if $x = y$.
- $d(x, y) = d(y, x)$, and
- $d(x, z) \leq d(x, z = y) + d(y, z)$.

The distance function is positive definite, symmetric and satisfies the triangle inequality. When $r > 0$, an **open ball of radius** $r$ centered at $x \in X$ is the set $B_r(x) = \{y \in X | d(x, y) < r\}$. In a metric space every open set can be written as a union of open balls.

5. A **continuous function** $(Y, \mathcal{V}) \xleftarrow{f} (X, T)$ is a function such that for every open set $V \in \mathcal{V}$, the inverse image $f^{-1}(V)$ is open in $X$.

6. A **homeomorphism** is a bijective (injective [or one-to-one] and surjective [or onto]) function that is continuous and that has a continuous inverse. A function is **injective** if $f(x) = f(y)$ implies that $x = y$. A function is **surjective** if $(\forall y \in Y)(\exists x \in X)$ such that $f(x) = y$. 
