General Instructions. Write your name on only the outside of your blue book. Do not write upon this test sheet. Write neat complete solutions to each of the problems below. As a courtesy to me, please write the problem down before you attempt a solution. Label your solutions carefully. There are 171 points.

Try breakfast tacos: scrambled eggs, tomatoes, peppers, and cheese on soft corn tortillas.

1. Compute the following definite, indefinite and improper integrals (8 points each).

(a) \[ \int_{1}^{2} \frac{1}{x^2} \, dx \]

(b) \[ \int \cos(x) \sin^4(x) \, dx \]

(c) \[ \int x \ln(x) \, dx \]

(d) \[ \int 2 \sin(x) \cos(x) \, dx \]

(e) \[ \int_{0}^{4} \frac{dx}{\sqrt{9 + x^2}} \]

(f) \[ \int \frac{1}{(x + 2)(x - 5)} \, dx \]

(g) \[ \int_{1}^{\infty} \frac{dx}{x^{1.2}} \]

2. (10 points) Compute the area that lies between the curves: \( y = \sin(x) \) and \( y = \cos(x) \) for \( x \in [0, \frac{\pi}{2}] \).
3. (10 points) Find the volume of the tetrahedron, by considering the triangular cross sections perpendicular to the $x$-axis.

4. (10 points) Compute the volume obtained by rotating the region bounded by the curves $y = \sqrt{x}$, $y = 0$, $x = 1$ about the $x$-axis.

5. (10 points) A leaky bucket is being lifted 20 meters. It originally holds 10 kilograms of water, and at the end of its journey it holds 5 kilos. How much work does it take to lift the bucket?
6. Determine which of the following sequences converge. If the sequence does converge, find the limit (5 points each).

(a) \(a_n = \left(1 + \frac{3}{n}\right)^n\)

(b) \(a_n = \frac{10^n}{n!}\)

7. Sum the series (5 points each):

(a) \(\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{(n+1) \cdot (n+2)} + \cdots\)

(b) \(1 + \frac{1}{3} + \frac{1}{9} + \cdots + \frac{1}{3^n} + \cdots\)

8. Use any test that you like to determine if the given series converges (5 points each)

(a) \(\sum_{n=1}^{\infty} \frac{1}{n!}\)

(b) \(\sum_{n=1}^{\infty} \left(1 + \frac{3}{n}\right)^n\)

(c) \(\sum_{n=1}^{\infty} \frac{n!}{(2n)!}\)

9. Compute the interval of convergence (remember to test end points!) for the series (10 points each):

(a) \(\sum_{n=0}^{\infty} \frac{x^n}{4^n}\)

(b) \(\sum_{n=1}^{\infty} \frac{x^n}{(3n)!}\)

10. (5 points) Compute the parametrization of the line segment from \((-3, 4)\) to \((8, 7)\) for \(t \in [0, 1]\).
11. (5 points) Give a parametrization of the ellipse \( \frac{x^2}{25} + \frac{y^2}{9} = 1 \) that starts at \((5, 0)\) and travels twice counter-clockwise in an interval \( t \in [0, 1] \).

12. (10 points) Compute the area enclosed by the polar graph for \( \theta \in [0, 2\pi] \):

\[
r = 1 + \cos(\theta).
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