

Math 115 Carter Final Exam Fall 2008

General Instructions:

I have enjoyed having each of you in my class. In particular, I am looking forward to seeing the progress that you have made over the course of the semester. Good luck on this exam!

Write your name on only the outside of your blue book. Do all of your work inside your blue book. Please do not write on this sheet. Please remove your hat. Put your test paper inside your blue book as you leave. Write neat complete solutions to each of the following problems.

To cook popcorn on an electric stove you need a good ear, good sense of smell, and an appropriately thick pan. Be prepared with an assistant to open the lid of the pan as you tip the popcorn over the bowl. Tipping the pan after the corn has popped prevents it from burning on the hot metal.

1. (5 points) Solve the inequality: $4 < |x + 2|$.
2. (5 points) Determine the equation of the line that passes through $(3, -2)$ and is parallel to the line $2x + y = 3$.
3. (5 points) Determine the equation of the line that passes through $(32, 0)$ and $(212, 100)$.
4. (10 points) Complete the square and sketch the graph of $f(x) = -x^2 + 4x + 6$. Determine the location of the vertex, the y -intercept, and x -intercepts (if any).
5. (10 points) Sketch the graph of the function $f(x) = |2x - 4| - 8$. Determine the location of the vertex, the y -intercept, and x -intercepts (if any).
6. (5 points) Determine the compositions $(f \circ g)(x)$ and $(g \circ f)(x)$ where $f(x) = x^2 + 1$ and $g(x) = x - 1$.

7. (10 points) Use the techniques of stretching and shifting to sketch the graph of $f(x) = 3x^3 - 2$. Indicate the y -intercept and the x -intercept(s).
8. (10 points) Sketch the graph of $f(x) = (x - 2)(x + 2)(x - 3)$.
9. (10 points) Factor the polynomial $f(x) = x^3 - 3x^2 + 4$ completely.
10. (10 points) Sketch the graph of $f(x) = \frac{x-3}{x+1}$.

11. (10 points) Find all values t such that $0 \leq t \leq 2\pi$ and for which

$$\sin(3t) = \frac{-\sqrt{2}}{2}.$$

12. (10 points) Sketch the graph of $y = -3 \sin(2x)$.
13. (10 points) Sketch the graph of $y = \tan\left(\frac{\pi x}{2}\right)$.
14. (10 points) By starting on the left-hand-side, prove the identity:

$$(1 - (\cos(x))^2)(\sec(x))^2 = (\tan(x))^2$$

15. (10 points) In a right triangle let A be the vertex with angle α , the vertices B and C have the analogous relationship with β and γ , respectively. Suppose that $\alpha = \pi/6$, B is a right angle, and that $BC = 15$. Determine the lengths of the remaining two sides AB and AC .
16. (10 points) Determine the interest rate that is needed for an initial investment of \$15,000 to grow to \$200,000 in a period of 18 years under continuously compounded interest. You may leave your answer in terms of a logarithm.
17. (10 points) Complete the squares, identify the quadratic curve, and sketch the graph.

$$y^2 - 4x^2 - 36y + 16x - 16 = 0$$