General Instructions. Write your name on only the outside of your blue books. Do not write on this test sheet, do all of your work inside your blue books. Write neat complete solutions to each of the problems in the blue book. Please put your test sheet inside the blue book as you leave. There are 155 points. Eat meals with family and friends.

1. (20 points) Find equations of the tangent plane and normal line to $w = 0$ level surface of $f(x, y, z) = x^2 y + xy^3 - z$, at the point $(2, 1, 6)$.

2. For the function $f(x, y) = x^3 y^4$,
   (a) (5 points) Find $\nabla f$
   (b) (10 points) Find the directional derivative of $f$ at the point $P_0$, $(2, -1)$, in the direction $v = (2, 1)$.
   (c) (10 points) Find the maximum rate of change of $f$ at the point $P_0$, and state the direction in which it occurs.

3. (a) (10 points) Find and classify the critical points of the function $f(x, y) = 4x - 3x^3 - 2xy^2$.
   (b) (20 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = xy$, subject to the constraint $4x^2 + 9y^2 = 32$.

4. (20 points) Evaluate the integral
   \[
   \int_{(1,1,1)}^{(2,2,2)} \frac{1}{y} dx + \left( \frac{1}{z} - \frac{x}{y^2} \right) dy - \frac{y}{z^2} dz.
   \]

5. (20 points) Use Green’s Theorem to evaluate
   \[
   \oint_C y^2 dx + x^2 dy
   \]
   where $C$ is the unit square $\{(x, y) : 0 \leq x \leq 1, \ 0 \leq y \leq 1\}$. 

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6. (20 points) Compute the line integral

\[ \frac{1}{2} \int_{\gamma} (y \, dx - x \, dy) \]

when \( \gamma \) is the straight line segment that starts at \((a, b)\) and ends at \((c, d)\).

7. Consider the vector field

\[ \vec{F} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\rho^3} \]

where \( \rho = \sqrt{x^2 + y^2 + z^2} \).

(a) (5 points) Compute \( \frac{\partial \rho}{\partial x} \).

(b) (5 points) **Use symmetry** to give the values for \( \frac{\partial \rho}{\partial y} \) and \( \frac{\partial \rho}{\partial z} \).

(c) (5 points) Use the results above to compute \( \frac{\partial}{\partial x}(x\rho^{-3}) \), and again symmetrically compute \( \frac{\partial}{\partial y}(y\rho^{-3}) \) and \( \frac{\partial}{\partial z}(z\rho^{-3}) \).

(d) (5 points) Use the divergence theorem to compute

\[ \int \int_F \vec{F} \cdot \vec{n} \, d\sigma \]

where \( F \) is the boundary of the domain \( D = \{(x, y, z) : a^2 \leq x^2 + y^2 + z^2 \leq b^2\} \) \( \vec{n} \) is the outward pointing normal, and \( d\sigma \) denotes the surface area form.

(e) (5 points) Now compute the outward flux of \( \vec{F} \) across the surface of a ball of radius \( a \) that is centered at the origin. Specifically, compute

\[ \int \int_S \vec{F} \cdot \vec{n} \, d\sigma \]

where

\( S = \{(x, y, z) : x^2 + y^2 + z^2 = a^2\} \)

whose outward pointing unit normal is \( \vec{n} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{a} \).