1. Let $X$ denote a set. Let $\mathcal{P}(X) = \{A \subset X\}$. Suppose that $\mathcal{T} \subset \mathcal{P}(X)$. Define the term $\mathcal{T}$ is a topology on $X$.

2. In case $X$ is finite, say $X$ has $n$ elements. Indicate why $\mathcal{P}(X)$ has $2^n$ elements.

3. Suppose that $X$ is infinite. Let $\mathcal{T} = \{A \subset X| X \setminus A \text{ is finite}\}$. Here $X \setminus A = \{x \in X| x \notin A\}$. Show that $\mathcal{T}$ is a topology on $X$. (If you have trouble with this because you could not solve #1, please ask for the definition.)

4. Suppose $(X, \mathcal{T})$ and $(Y, \mathcal{V})$ are topological spaces and $Y \xrightarrow{f} X$ denotes a function. Define $f$ is continuous on $X$.

5. Describe the open sets in the usual topology on the real line $\mathbb{R}$. This is the topology induced by the metric $d(x, y) = |y - x|$ on $\mathbb{R}$.

6. Consider the identity function on $\mathbb{R}$. Also consider the usual and finite complement topology (problem #3). The identity function is continuous in one direction. Which one?

7. Let $X$ denote a set; and $A \mapsto \overline{A}$ a function from $\mathcal{P}(X)$ to $\mathcal{P}(X)$ such that
   
   (a) $\emptyset = \emptyset$;
   (b) $A \subset \overline{A}$;
   (c) $\overline{\overline{A}} = \overline{A}$;
   (d) $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

   Show that $\mathcal{T} = \{X \setminus \overline{A}| A \in \mathcal{P}(X)\}$ is a topology on $X$. 

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