I am sorry that this appears so late. I had a kitchen emergency this weekend that took longer than I anticipated. This study guide is edited from past study guides.

1. **This problem will appear on the test!** What does the expression
   \[ \lim_{x \to c} f(x) = L \]
   mean? State the definition of a limit.

2. **This problem will appear on the test!** Give a proof that
   \[ \lim_{h \to 0^+} \frac{\sin(h)}{h} = 1. \]

3. Use the fact that \( \lim_{h \to 0} \frac{\sin(h)}{h} = 1 \), the Pythagorean identity, and elementary algebra to prove that
   \[ \lim_{h \to 0} \frac{\cos(h) - 1}{h} = 0. \]

4. **There will be a problem comparable to this on the test!** Determine the \( y \)-intercept, and use the technique of completing the square to determine the location of the vertex, to determine the \( x \)-intercepts (if any) and sketch the graph.
   \[ y = 3x^2 - 15x - 25, \]

5. Determine the \( x \) and \( y \)-intercepts, the horizontal and vertical asymptotes and sketch the graph of the linear fractional transformation:
   \[ y = \frac{2x - 6}{x + 5}. \]

6. Sketch the graph of
   \[ f(x) = 2\sin(3x) \]
   over two full periods.

7. A certain radioactive isotope has a half-life of 5287 years. If 3 grams of the isotope are initially present, how long do you expect it will take until 0.5 grams remain. Leave your answer in terms of a natural logarithm.
8. Use the rules for computing the limit for each of the following problems.

(a) \[ \lim_{x \to 3} \frac{x^2 - 9}{x - 3} \]

(b) \[ \lim_{h \to 0} \frac{\frac{1}{5+h} - \frac{1}{5}}{h} \]

(c) \[ \lim_{x \to \infty} \frac{3x^2 - 5}{2x^2 + 2x - 8} \]

(d) \[ \lim_{\theta \to 0} \frac{\tan (4\theta)}{\sin (6\theta)} \]

(e) \[ \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \]

(f) \[ \lim_{x \to -3} \frac{x + 3}{x^2 + 4x + 3} \]

(g) \[ \lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 3x + 2} \]

(h) \[ \lim_{x \to 0} \frac{\sin (2x)}{\sin (3x)} \]

(i) \[ \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \]

(j) \[ \lim_{x \to -3} \frac{2 - \sqrt{x^2 - 5}}{x + 3} \]

(k) \[ \lim_{x \to 0} \frac{x \csc (2x)}{\cos (5x)} \]

(l) \[ \lim_{y \to 0} \frac{\sin (3y) \cot (5y)}{y \cot (4y)} \]

(m) \[ \lim_{x \to a} \frac{x^5 - a^5}{x - a} \]