1. Recall that the usual topology on $\mathbb{R}^n$ has as a basis the set of open balls

$$B_\epsilon(\bar{x}) = \left\{ \bar{y} \in \mathbb{R}^n : \sqrt{\sum_{i=1}^{n} (y_i - x_i)^2} < \epsilon \right\}$$

where $\epsilon > 0$ and $\bar{x} = (x_1, x_2, \ldots, x_n)$ ranges over all points in $\mathbb{R}^n$.

In $\mathbb{R}^2$ consider the set

$$D_\delta(\bar{x}) = \left\{ \bar{y} \in \mathbb{R}^2 : \sum_{i=1}^{n} |y_i - x_i| < \delta \right\}.$$  

(a) Show $D_\delta(\bar{x})$ is open in the usual topology for $\mathbb{R}^2$.

(b) Let $\mathcal{D}$ denote the topology with basis

$$\{D_\delta(\bar{x}) : \delta > 0 \ & \bar{x} \in \mathbb{R}^2\}.$$  

Show $B_\epsilon(\bar{x})$ is open in $\mathcal{D}$.

2. Consider

$$S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1 \ & \ y \neq 0\}$$

as a subset of $\mathbb{R}^2$ in the usual topology.

(a) Is the set connected? Prove your assertion.

(b) Is $\mathbb{R}^2 \setminus S$ connected?

(c) Is $S$ open in $\mathbb{R}^2$? Prove your assertion.

(d) Describe the closure of $S$.

3. Consider $\mathcal{C}$ to be the Cantor set as a subset of $\mathbb{R}$, and consider $\mathbb{Q} \cap [0,1]$ — the set of rational numbers in the unit interval.

(a) Is $\mathcal{C}$ closed?

(b) Is $\mathbb{Q} \cap [0,1]$ closed?

(c) Is $\mathbb{Q} \cap [0,1]$ discrete?

(d) Is $\mathcal{C}$ discrete?

(e) Are either $\mathcal{C}$ or $\mathbb{Q} \cap [0,1]$ connected?

(f) Are $\mathcal{C}$ and $\mathbb{Q} \cap [0,1]$ homeomorphic?

Justify your answers.