Peptalk for Calculus 1

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When I was a senior in high school, I did not make the soccer team. It was the second time in my high school career that I tried out for a team. I did not make the team as a junior, and at that time, I was told that I could try out again. Over 50 students tried out, and the team was cut down to the city's requisite size; I think 25. Because so many boys tried out, the coaches were brutal to us in practice. We all went home in the evening aching from toe to forehead. The coach was trying to cull the team of quitters. Very few quit. So the coach had to cut us on the basis of talent. I had none. I don't blame the coaches for not letting me play on the team. A senior who is no good is not entitled to a place on the team.

I tell you my soccer story because as your math professor I want you to excel in my class. Some of you may try hard and still not succeed. Of course, my ambition for you is not merely success, but excellence. Let me reiterate, *my goal is for* **you all** *to excel,* and I will work hard throughout the semester to teach you all that you need to know to excel in the course.

You may only see yourself as an individual in the class, and you may believe that your success or failure depends only on how well I communicate to you the methods and the algorithms necessary to work the problems. You, the individual, may not share *my* perception that you, the collection, constitute a class. Please understand that the class's performance as a whole depends on your individual performance. When each student works hard, the class excels. The class that you attend forms a team. By extension, I am not the opposition, but the coach. (But please don't call me "Coach." You may call me "Professor Carter," "Dr. Carter," "Professor Zap" or "Dude." I don't respond well to, "Heyya").

Yes, you are an individual. When you come to me for help, or as I grade your paper, I will treat you as a unique person with unique abilities. But you are also a member of this team that is my class. Not only is my goal for you as an individual to excel, but also my goal is for this class to be the best that I have ever taught. So I want you *all* to do your best, or even more: I want you all to master the techniques and ideas of Calculus 1 beyond what *you* originally thought possible.

Yes, I will do my best to communicate the mathematical ideas involved in calculus. But you must be aware that calculus is the beginning of college level mathematics. It is a new way of thinking. Most every college student before you had trouble learning calculus. It is not a particularly difficult subject, but it does present a new way of looking at things. And in order to excel at this course you have to learn and become comfortable with new ideas.

Not only that, but I will expect that you are fluent in many old ideas. I'll say more about that later.

Some parts of working a calculus problem are routine. But calculus in particular, and mathematics in general, is more than applying a sequence of steps, turning the crank and getting the answer. Each problem will present its own unique set of challenges. How do you learn to master a subject when each problem that is presented is new?

You work problems. You read the book. You attend class. Having attempted the problems, and having seen me work some (not all) of the problems, you try to emulate my solution. You take notes. You re-read your notes. You re-work the problems that confused you. You build a knowledge-base one example at a time, and you master as many examples as you can. You explain that which you understand to your peers. You go to your peers, the tutoring lab, and to me for help. You look through youtube videos when nothing else works.

It may not be sufficient to work through the homework only once! You may be one of those people like me who must work a given problem several times before it is mastered.

You also have to learn how to write your solutions clearly. I will look at how you write solutions. If you have not expressed yourself clearly, then I will get grumpy. You don't want your grader to be grumpy. I will not only look for correct answers, but I will also look at the method that you used to find your answer. If you are thinking clearly, then you will write clearly. The converse is also true. If your answer is incorrect, but your method is sound, then I want to point out to you where you first made a mistake. I can't do that if I can't read your solution. Furthermore, a substantial aspect of an answer is whether the answer is reasonable. For example, earthly distances will be somewhere between several inches to a couple of hundred miles. Monetary gain when interest accrues over small periods of time should be modest.

I cannot achieve the goal of your excellence if you do not share that goal. In order to excel at sports, music, or business, you have to keep trying. The same is true of mathematics.

There is a song I am learning to play on the guitar. It has a sequence of a particularly difficult (for me) chord changes. I practice that part of the song many times each day because I want to learn how to play it. What I am doing may not be enough, but if I don't practice this part of the song, I will never play it. Sometimes I try alternate fingerings. Sometimes I analyze the sequence. Just now, I thought through the sequence and positioned my left hand as if I were playing it. The principle of mastery through practice, repetition, and visualization applies to music, exercise, mathematics, and virtually anything that you want to learn.

If you do not want to learn calculus, then I really don't want to have you in my class because you and I will not have a shared goal. Your lack of progress will also hamper your peers. So leave, but don't go away angry.

Working a mathematics problem is a dynamic process. I write things on the board in the *order* that I write them because I want to emphasize the dynamics of the solution. I will emphasize this organization of ideas when I am writing these things. Watch. Watch and take notes. I am pretty good at working calculus problems. You can't learn from my methods unless you come to class. I will tell you things in person and in real time that the

author cannot tell you in his book.

But he thought long and hard as he wrote the book. He will tell you things that I cannot. Pay him the respect, not only of buying the book he wrote, but of reading it. Make his book your book by reading it and devouring its content. I cannot and will not teach you everything there is to know about the subject. Not only that, I cannot even cover every detail of the course and work all the problems in the time allotted. That is why you have a book. Read it, work problems, re-work problems, and come to class. Write neat, complete solutions.

Over the summer, I thought about my expectations for you. Here is a partial list of skills that I think you should have:

1. Be able to perform arithmetic — including but not limited to being able to add, subtract, multiply and divide real numbers, rational numbers, and elementary algebraic expressions. In particular, be able to add fractions.

2. Be able to simplify algebraic expressions — including but not limited to combining like factors in numerators and denominators into a single factor with a positive or negative exponent.

3. Be able to solve linear equations in one unknown; in particular, rearrange a complicated expression in which the unknown appears multiple times on either side with a variety of positive or negative coefficients.

4. Be able to factor quadratic expressions and apply the quadratic formula.

5. Be able to graph a linear equation and to determine the equation of a line in a variety of forms.

6. Be able to solve basic problems with proportionalities, unit conversions, and percentages.

7. Be able to sketch the graph of a circle or a parabola by completing the square.

8. Be able to determine the locus of the vertex of a parabola, the intecepts, and to solve a general quadratic equation.

9. Be able to sketch the graph of a function of the form $(y-d) = af(bx-c)$ having been given the graph of $y = f(x)$.

10. Have some familiarity with the graphs of specific polynomial and rational functions (*e.g.* polynomials that are factored or are easily factorable; rational functions whose numerator degrees and denominator degrees are less than two or three).

11. Be able to simplify logarithmic and exponential expressions.

12. Be able to solve growth and decay problems.

13. Be able to immediately give values for any trig function for angles 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$.

14. Be able to compute the trig functions in other quadrants if the angle is a integral multiple of $\frac{\pi}{6}$ or $\frac{\pi}{4}$.

15. Be able to convert between degrees and radians easily and seamlessly.

16. Be able to sketch the graphs of the form $y = Af(bx - c) + d$ where $y = f(x)$ is a trig function

17. Be able to solve triangles.

18. Be able to prove elementary trig identities.

19. Be able to solve trig equations up to degree 2.

20. Be able to apply trig identities to compute values of trig functions for unusual angles.

I am not naive. I know that you may not have total instant recall on all of these items. But as we go through the review chapter, you will relearn these skills. I will happily assist you on these items and anything else that I can as the course progresses. Good luck. Let us begin.