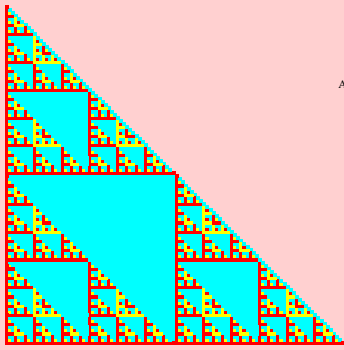
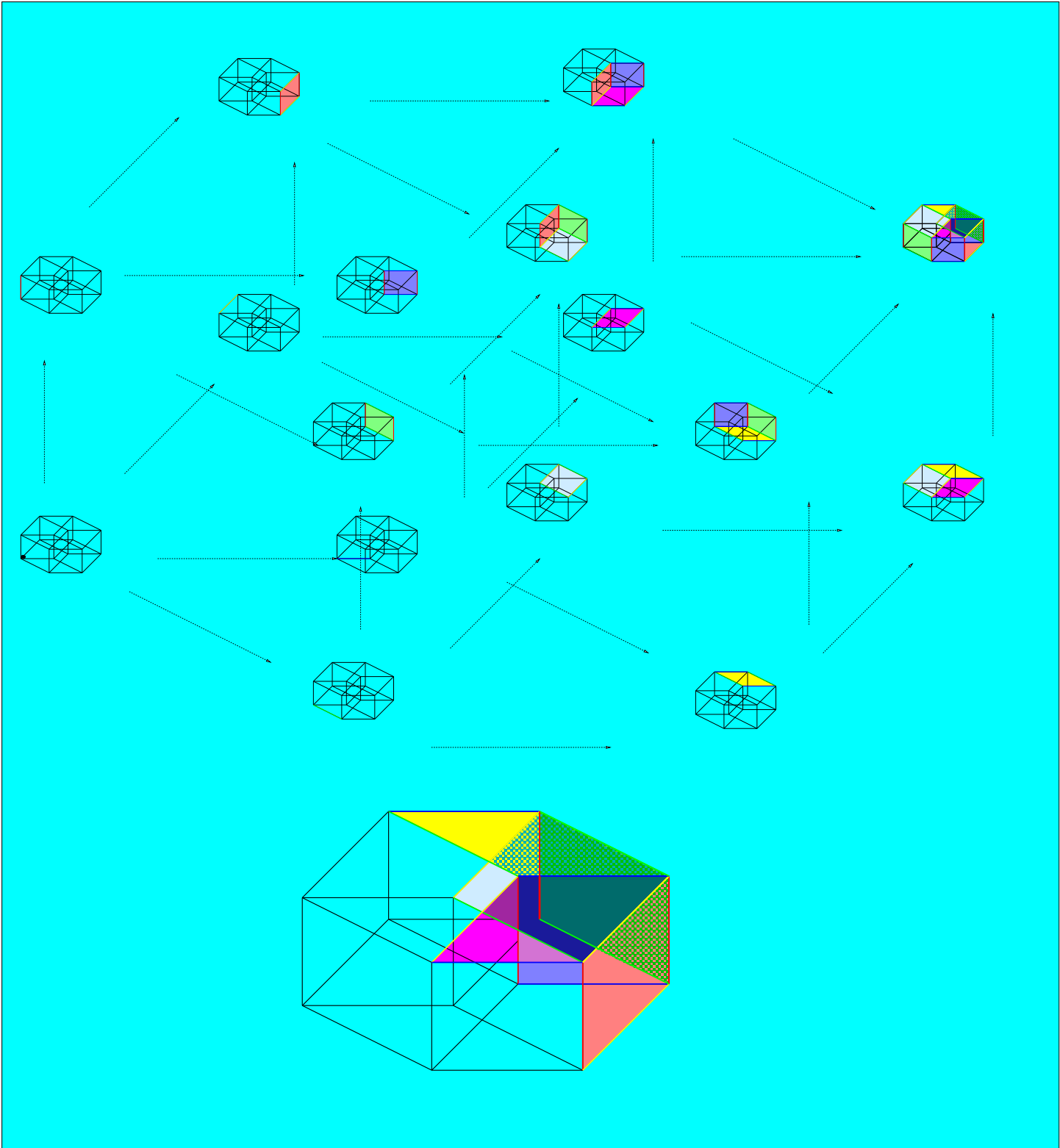
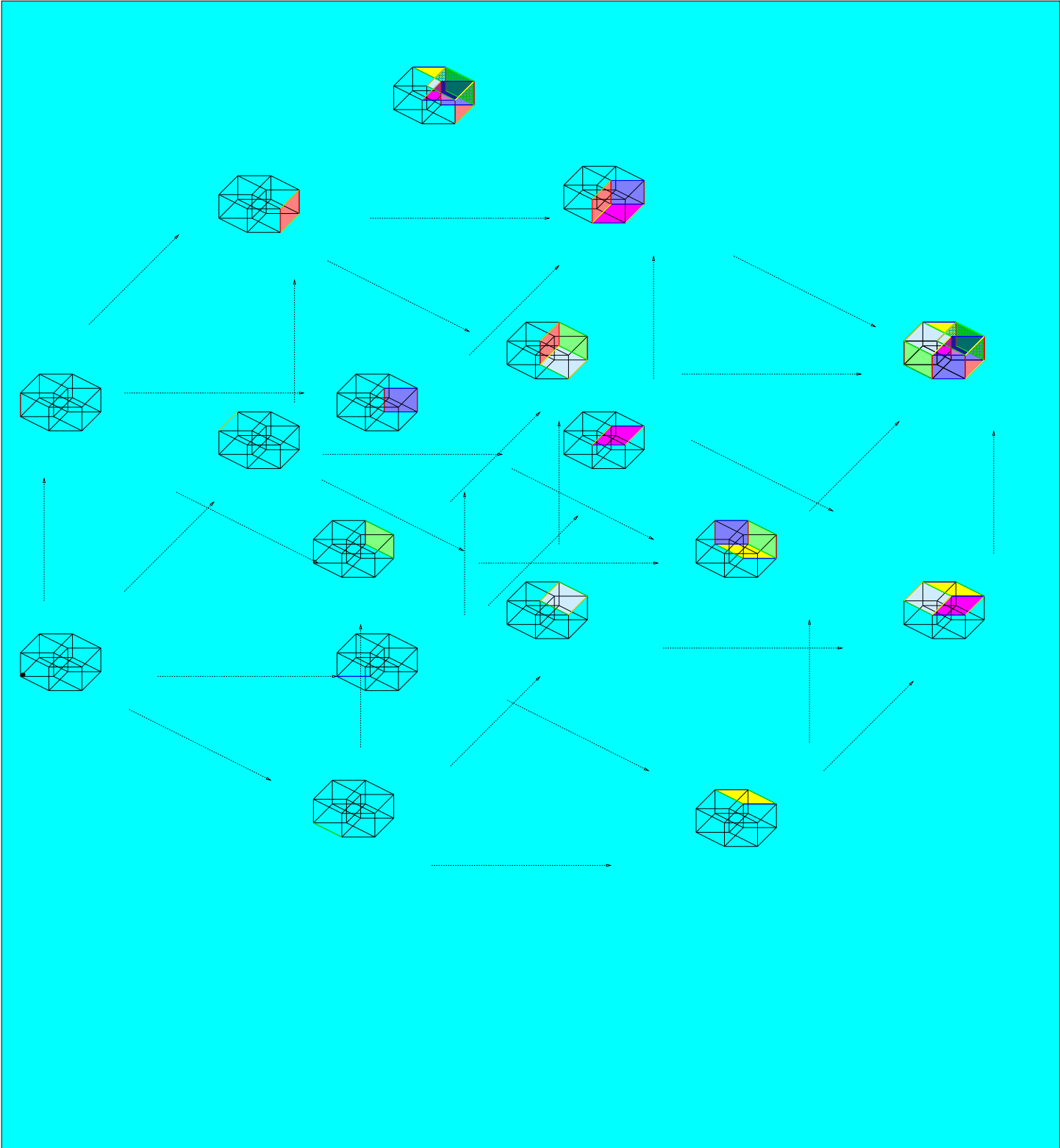
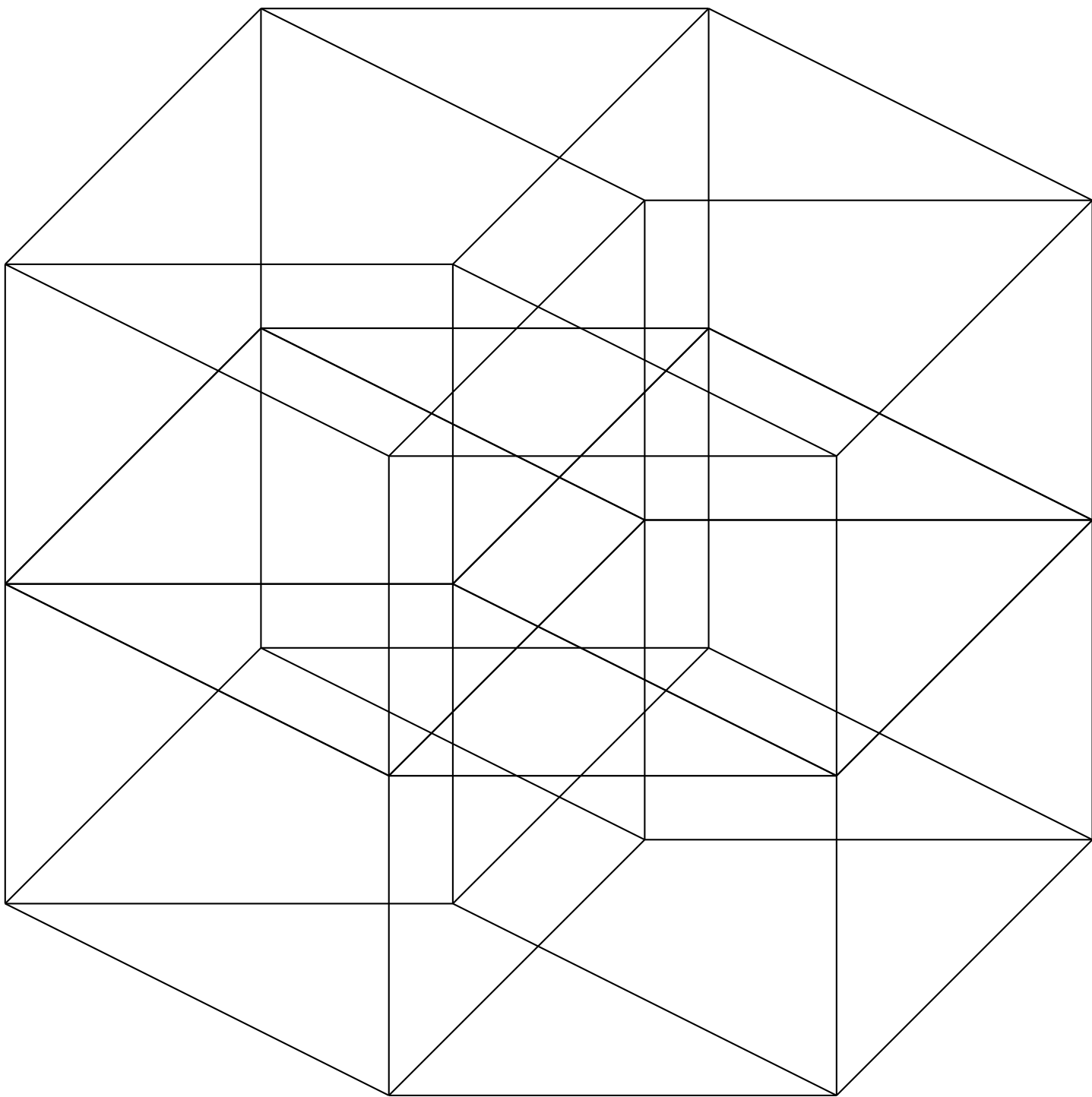


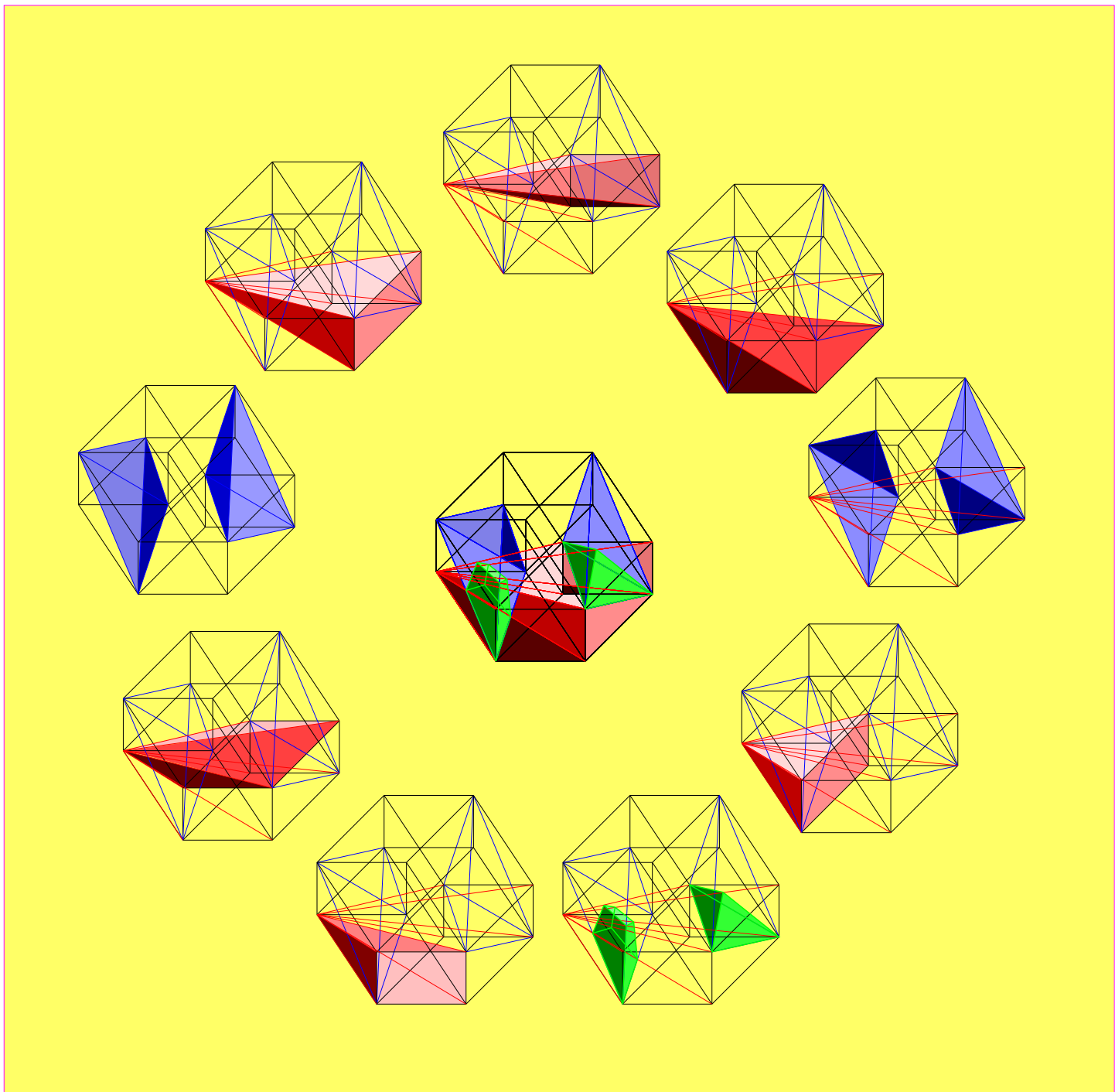
The Binomial Theorem
expresses the n th power of the sum, $A+B$, as the sum over all binary sequences of A s and B s. The power of the sum is the hyper-volume of the hypercube whose edge length is $A+B$. The binary sequence summands are each the hyper-volume of a hypercube whose edge lengths are some choice of A s and B s. Pascal's triangle tells us how many terms of each type.

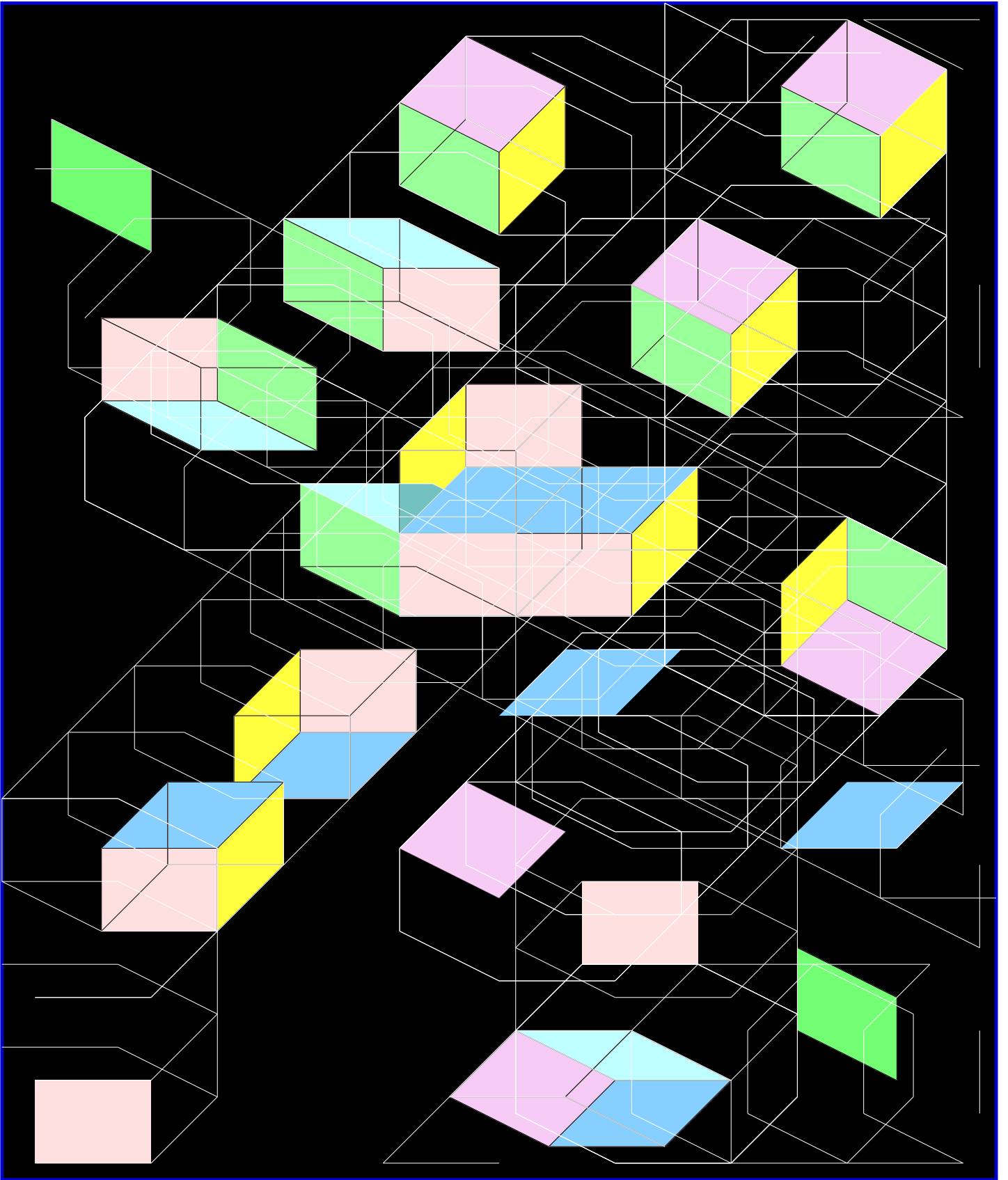


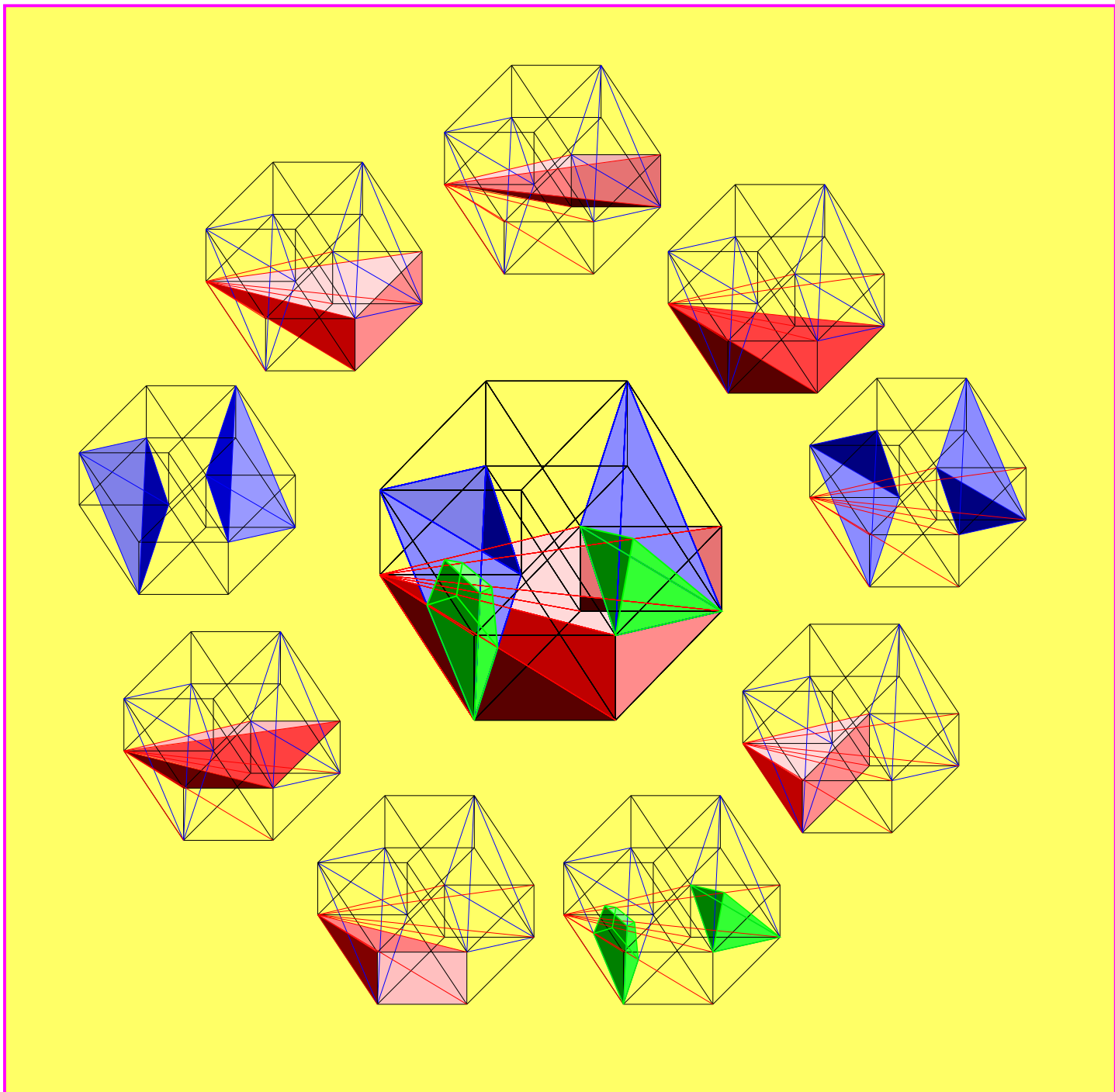


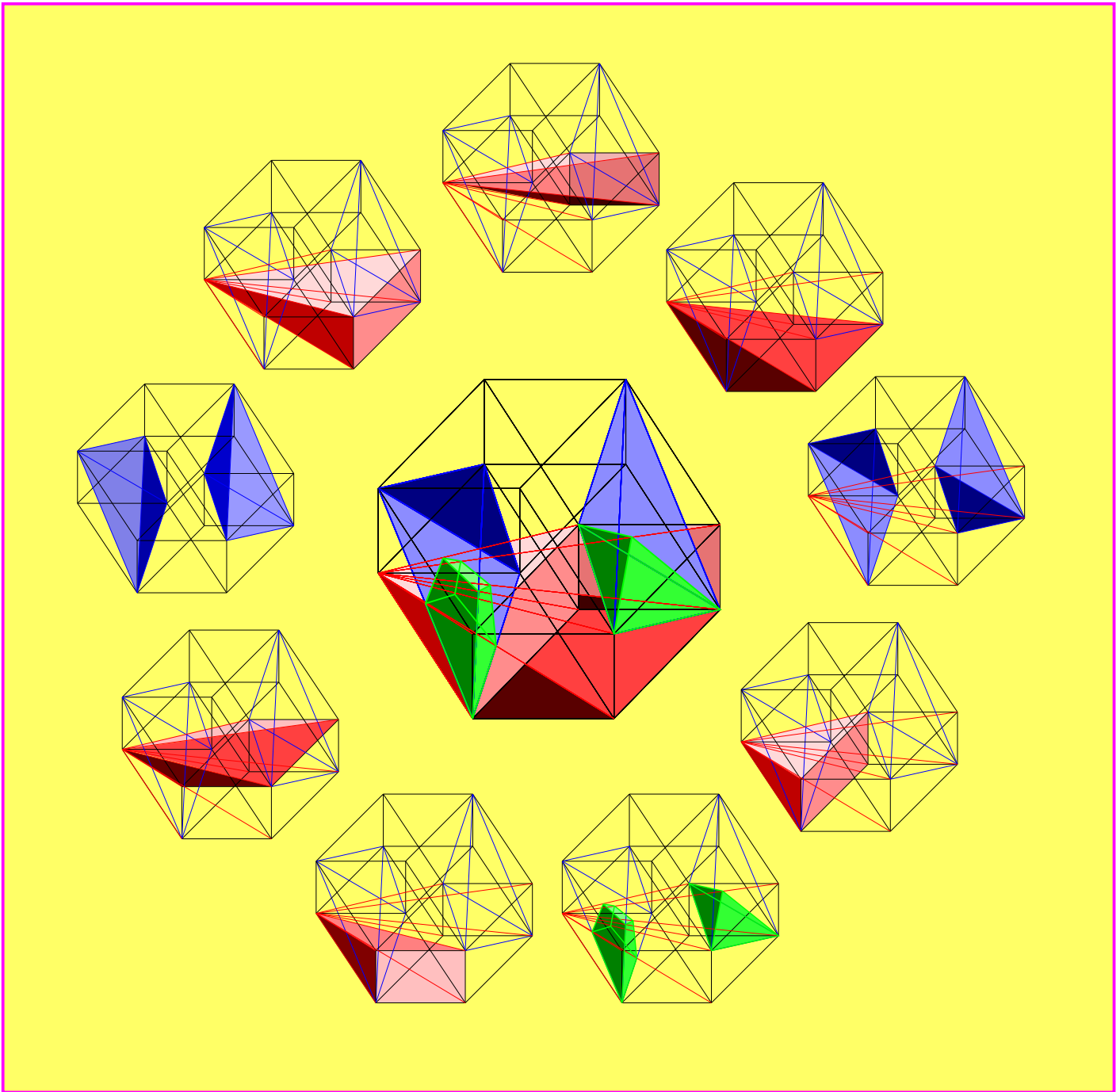




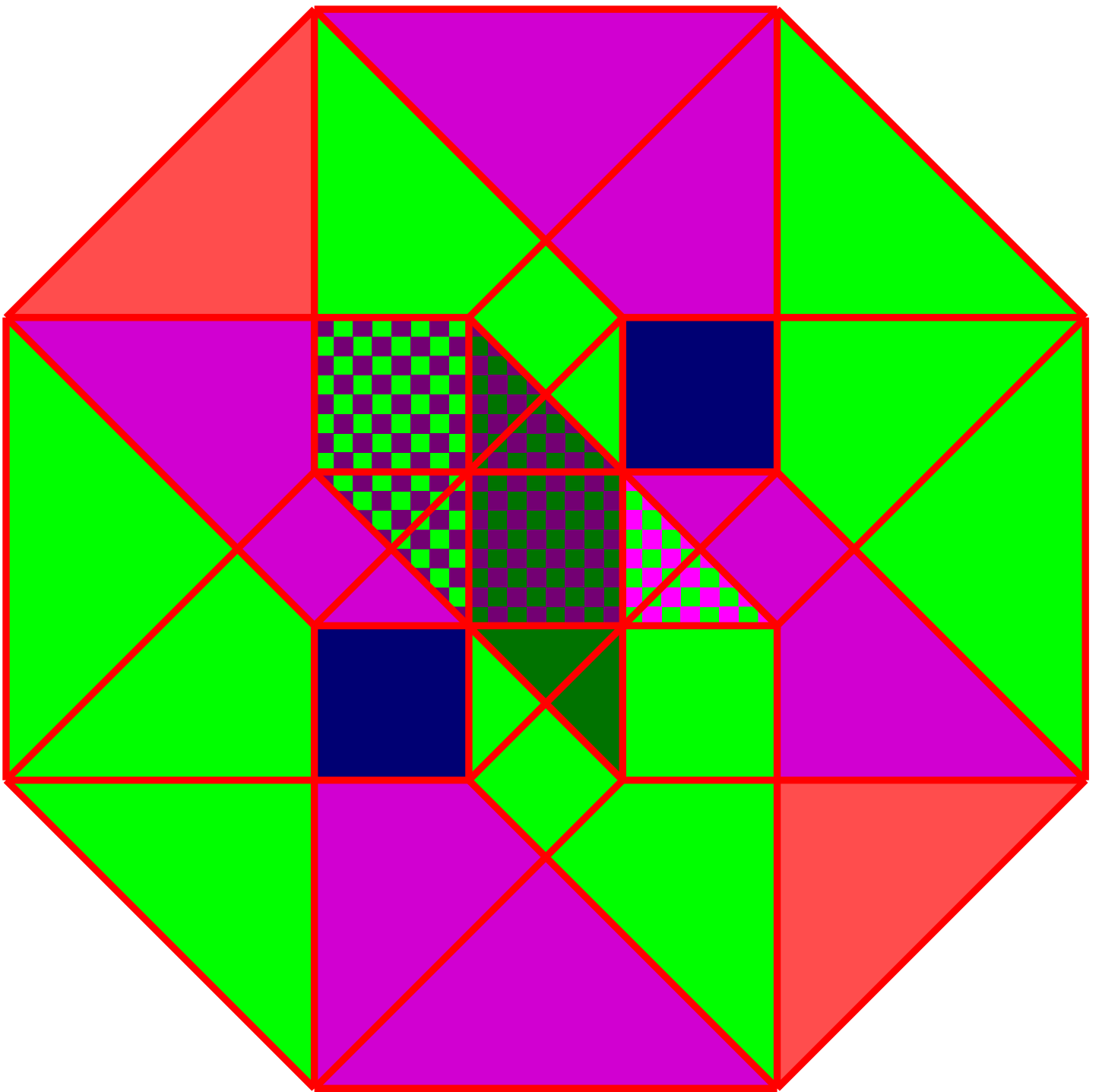


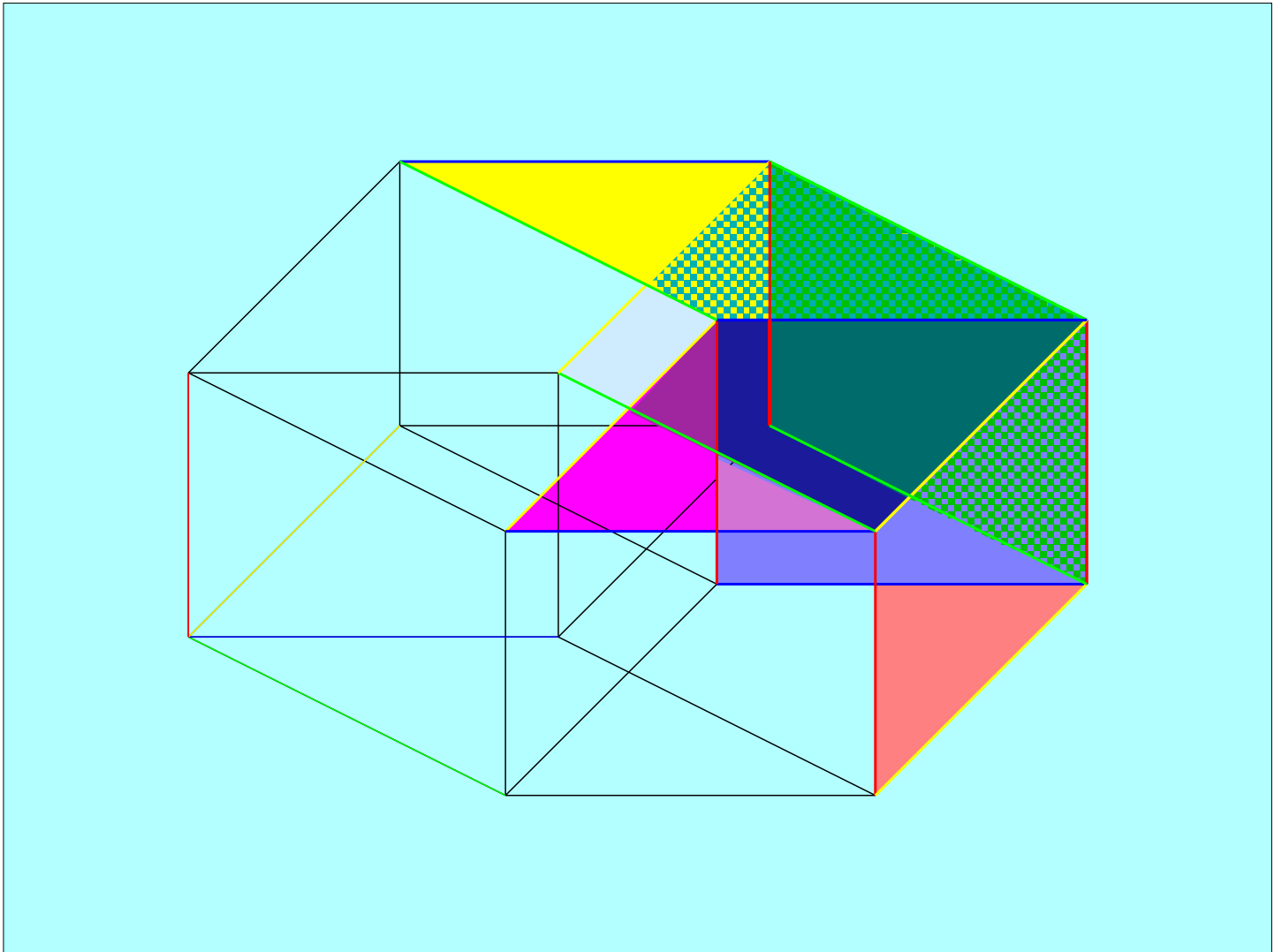


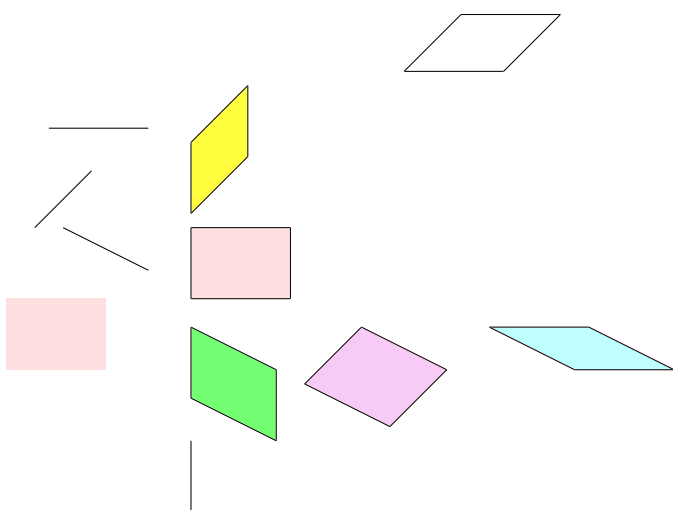
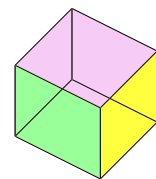
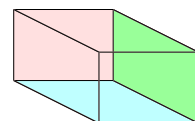
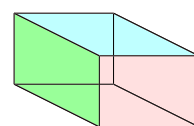
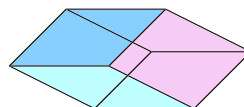
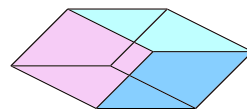
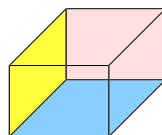
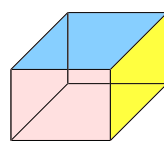
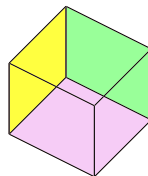
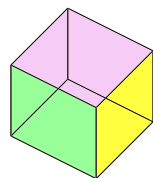
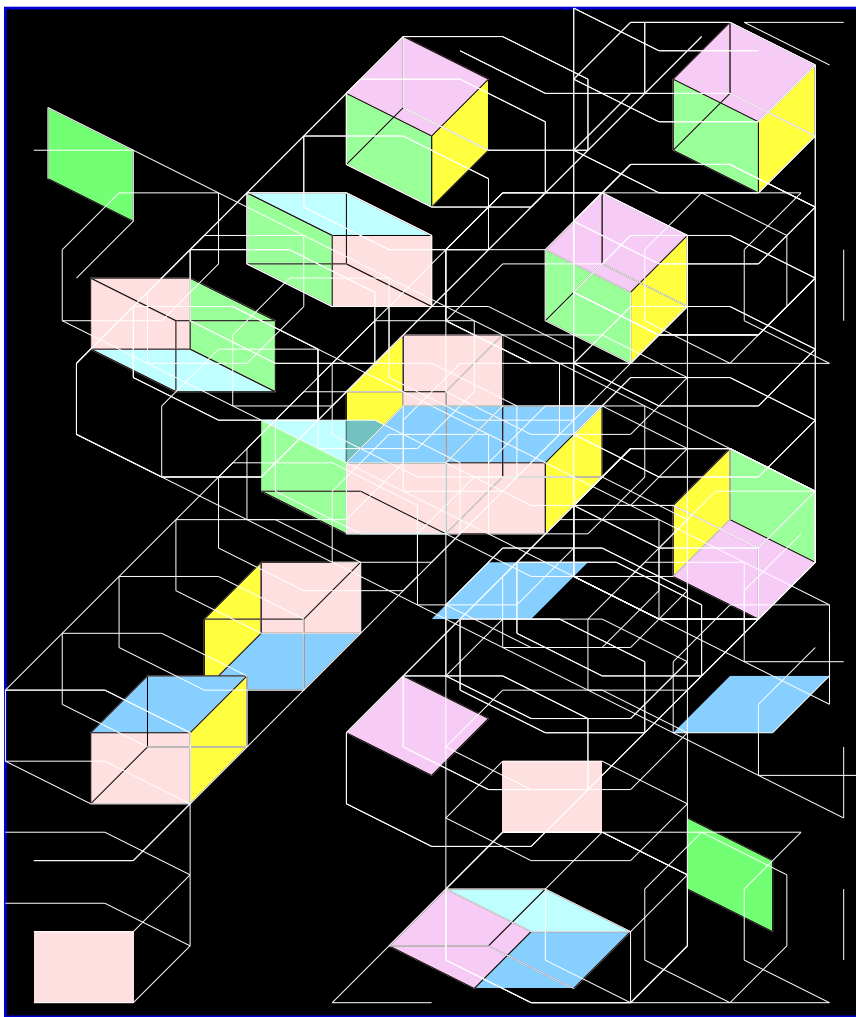


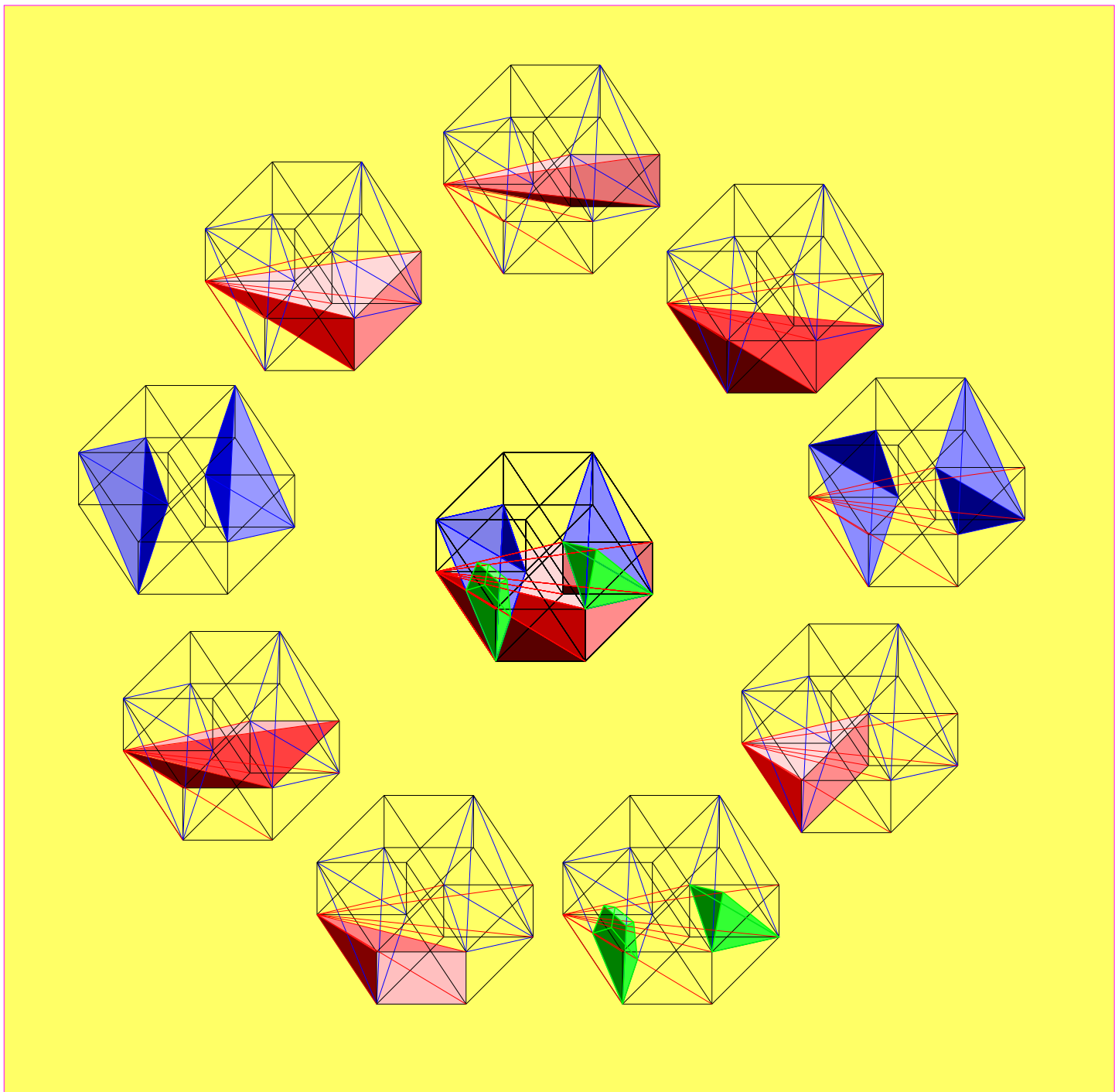


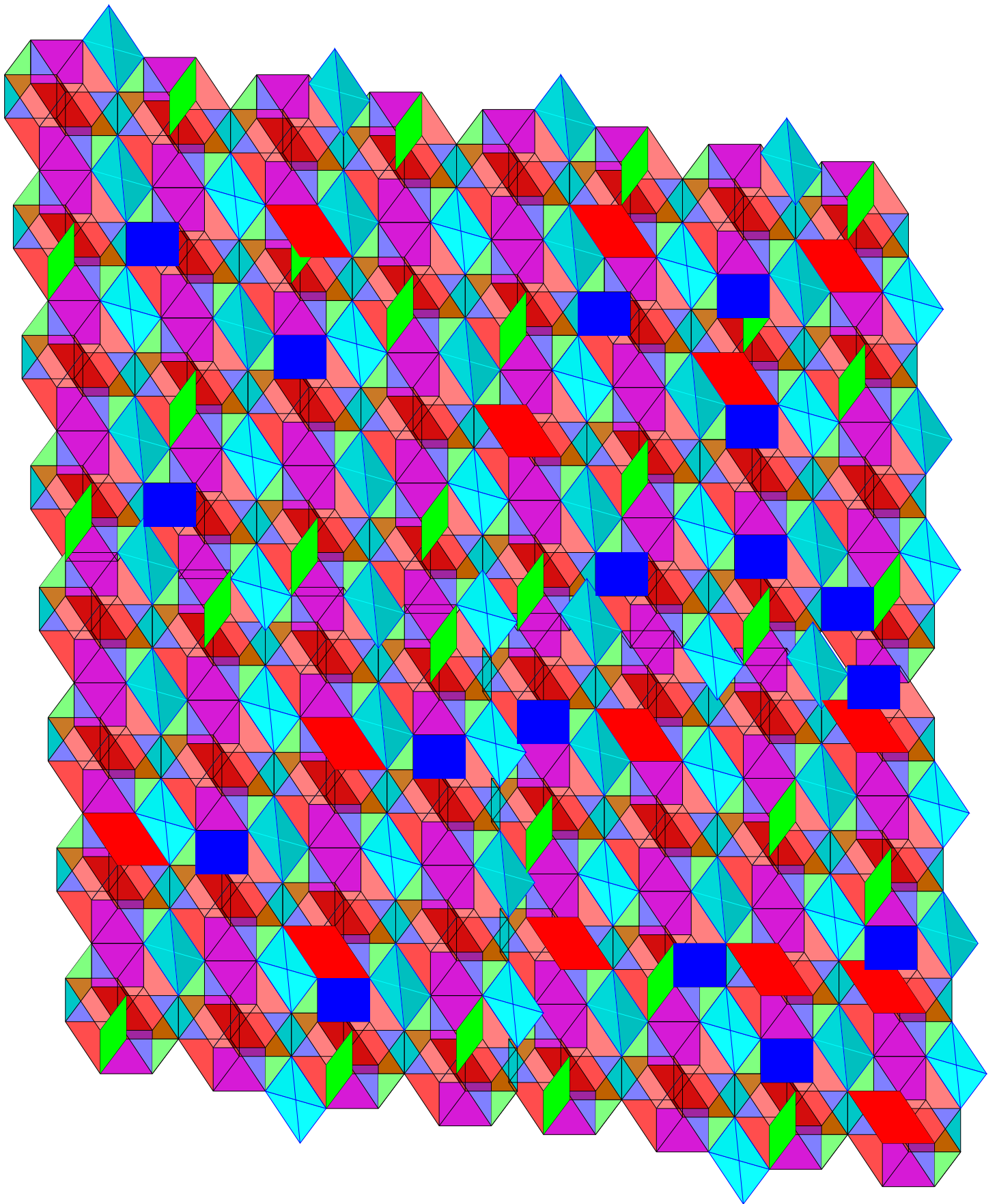
Inside a 4-dimensional cube, the 3-dimensional top cell $(x,y,z,1)$ is coned to the origin, $(0,0,0,0)$. The red pyramids indicate the six pyramidal cells on the various 2-dimensional faces of that cube. The blue tetrahedron on the left is the smallest convex set that contains the unit coordinate vectors. The blue tetrahedron on the right is the smallest convex set that contains the four points $(1,1,1,0)$, $(1,1,0,1)$, $(1,0,1,1)$, and $(0,1,1,1)$. The red cone intersects these two tetrahedra in green solids. On the left is an object that has 2-dimensional faces which are kites and which together form a cube-like object. On the right, four triangular faces form a tetrahedron. Either of these figures can be rotated and reflected within the 4-cube, so that four copies of the green objects tile the blue tetrahedron. Under such rigid motions, the cubical base of the red pyramid rotates to $(x,y,1,w)$ then to $(x,1,z,w)$ and finally to $(1,y,z,w)$. Meanwhile, the resulting four copies of the 4-dimensional pyramid fill the 4-cube. Thus the original cone occupies a quarter of the space in the 4-cube. Therefore, the sum of the volumes of cubes whose edge lengths range from 0 to 1 is $1/4$. Analogous phenomena occur in all dimensions.

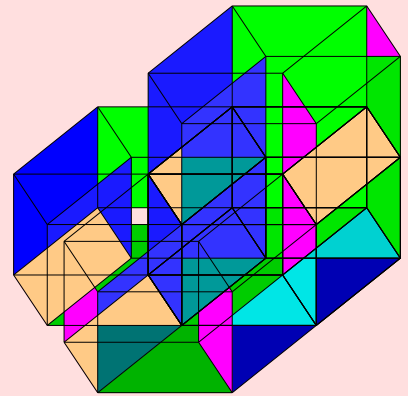
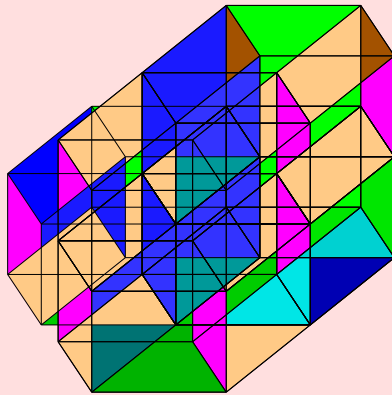
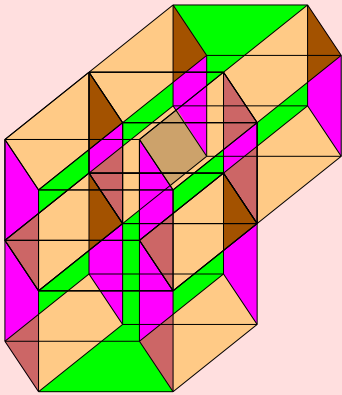


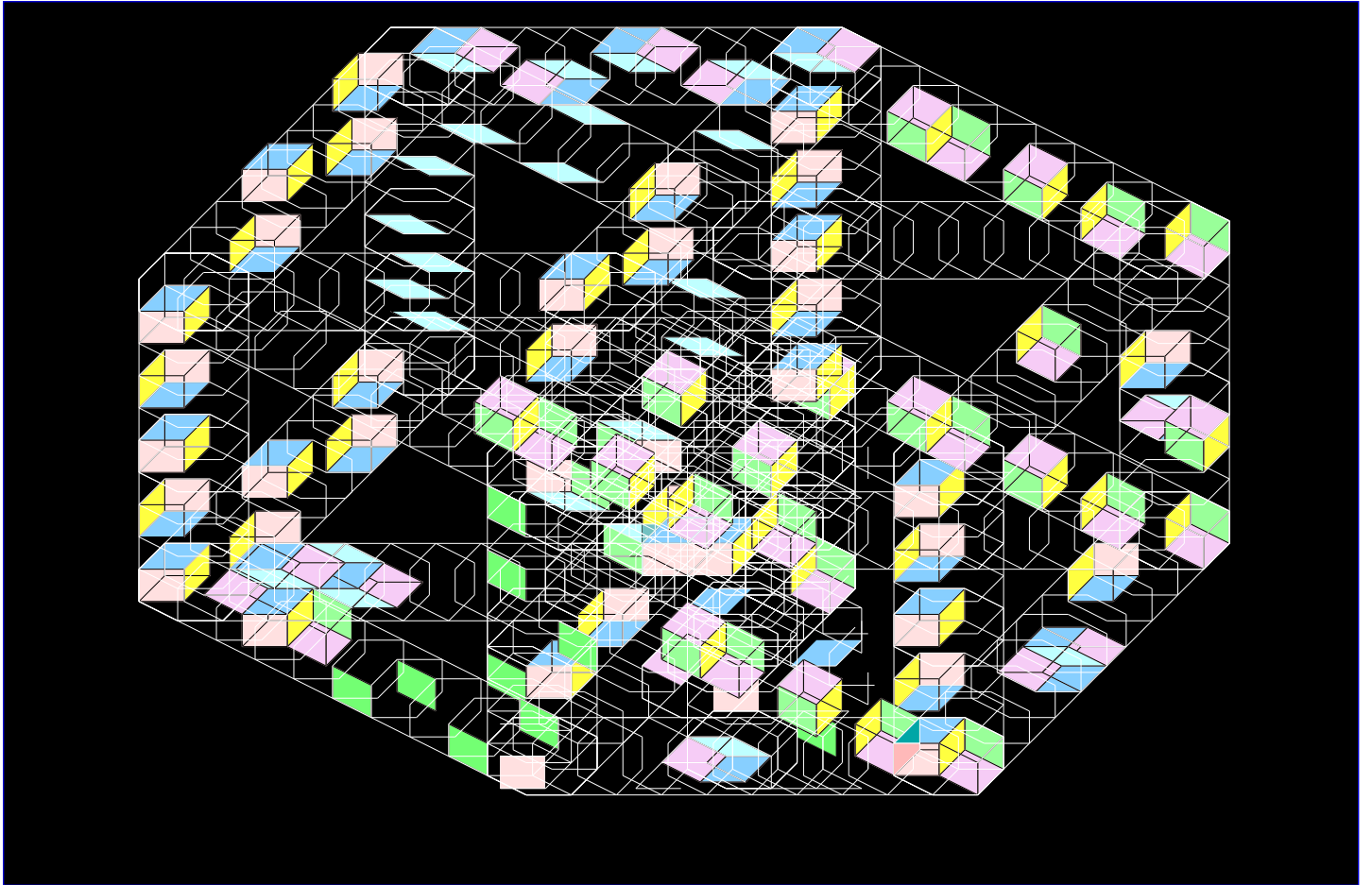


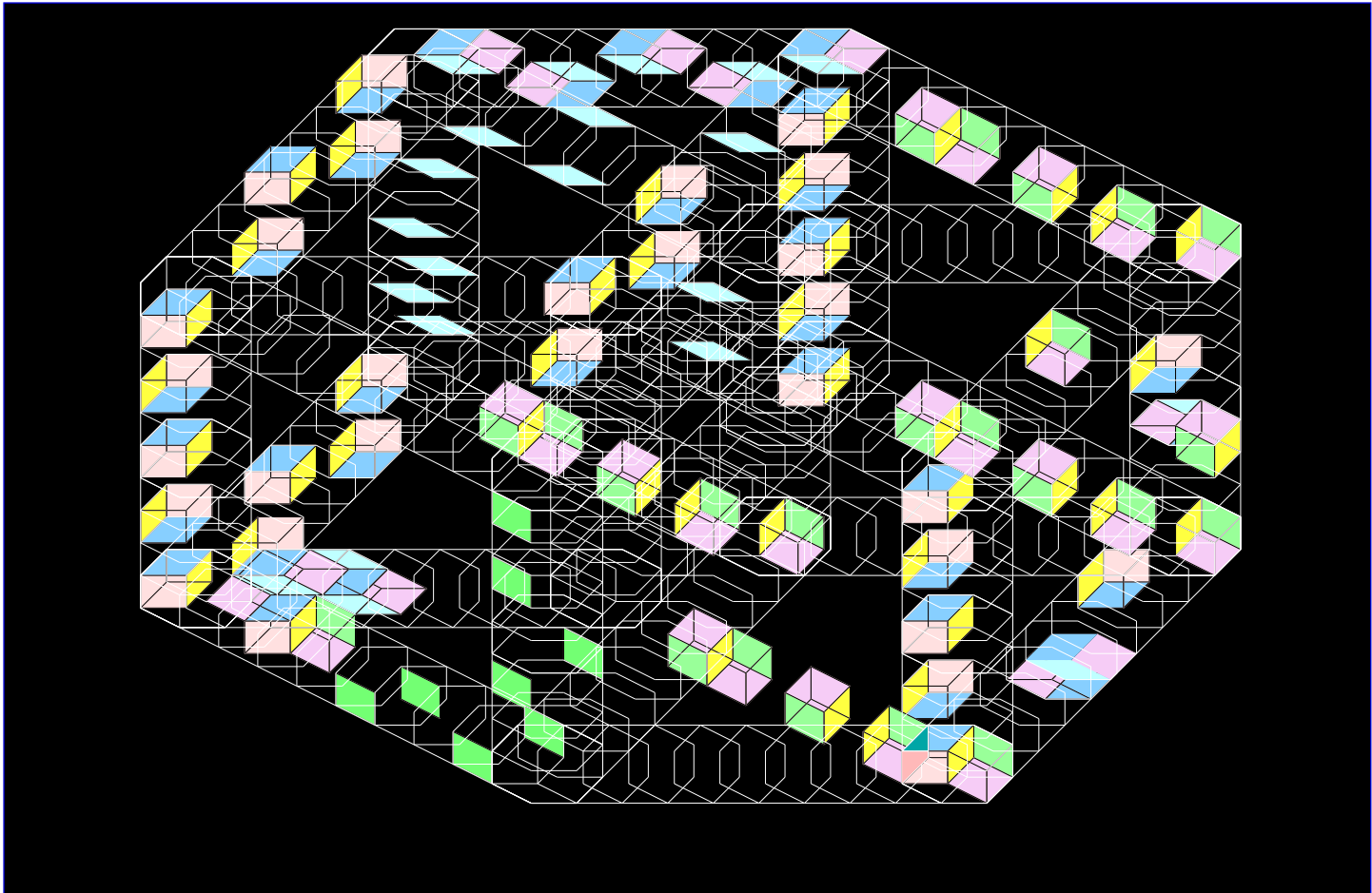


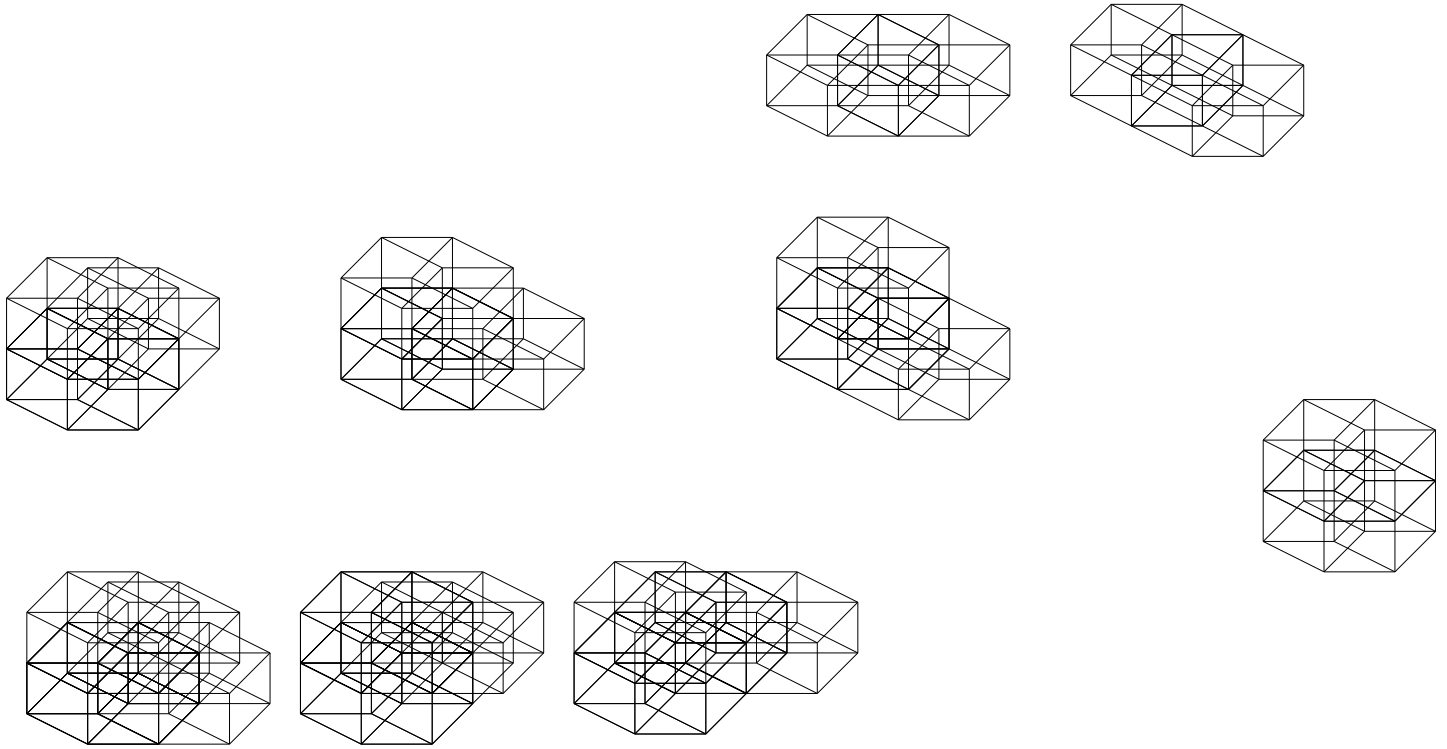












123

