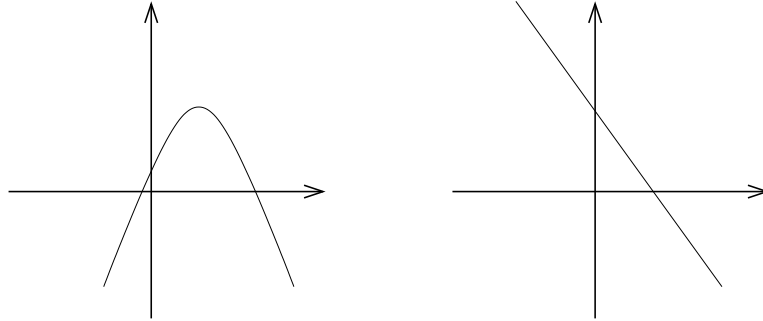


# Math 125    Carter    Final    Spring 2003

Do all of your work in your blue book. Don't write on the test, we won't look at it. Write your name on only the outside of your blue book. Thank you for your hard work in this course. Do well!

1. *5 points* In the Figure below one of the graphs represents a function; the other represents its derivative. Which is which and why?



2. Compute  $f'(x)$  or  $\frac{dy}{dx}$  derivatives of the following (*5 points each*):

(a)  $f(x) = -x^2 + 4x + 5$

(b)  $y = \sin(2x)$

(c)  $y = \sqrt{(x^3 + 3x - 4)}$

(d)  $f(x) = e^{x^2+1}$

(e)  $f(x) = \ln(\cos(x))$

(f)  $xy - y^3 = 5$

3. Compute the limit of the Newton quotient for the function *5 points*:

$$f(x) = \sqrt{x - 2}$$

4. Compute the following limits *5 points*:

(a)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

(b)  $\lim_{x \rightarrow 9} \frac{x^2 - 81}{x - 9}$

(c)  $\lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 - 5x + 13}{34x^4 + 18x^3 - 2x^2 - 405x - 37}$

5. *5 points* Prove by induction,

$$1^2 + 2^2 + \dots + N^2 = \frac{(N)(N + 1)(2N + 1)}{6}.$$

6. *10 points* A projectile is shot upward from the edge of a 20 meter cliff and moves vertically along a straight line according to the equation,

$$s(t) = -5t^2 + 45t + 20$$

where  $t \geq 0$  is measured in seconds, and the vertical position,  $s$ , is measured in meters.

- (a) Sketch a graph of the position as a function of time. Include an appropriate domain.
  - (b) When does the projectile reach its highest point?
  - (c) What is the velocity of the projectile as it hits the ground ( $s(t) = 0$ )?
  - (d) When does the projectile pass the edge of the cliff?
7. *10 points* Sketch the graph of the function

$$f(x) = \frac{1}{x^2 + 1},$$

include critical point(s), inflection points, and any asymptotic behavior.

8. *10 points* Sand falls in a conical pile at a constant rate of 3 cubic meters per second. The radius of the cone is always twice the height. How fast is the height increasing when the height is 4 meters? (Hint: The volume of a cone is given by  $V = \frac{\pi}{3}r^2h$ )
9. *10 points* Show that among all the rectangles that are inscribed in a circle of radius 1 the rectangle that has the maximum area is a square. Hint: Use a cartesian coordinate system.
10. *10 points* Use your calculator to ESTIMATE

$$\int_1^2 \ln x \, dx$$

By subdividing the interval  $[0, 1]$  into 5 subintervals of length  $1/5$ .

11. *10 points* Compute the equation of the line tangent to the curve,  $y = \cos(x)$  at the point  $(\pi/6, \sqrt{3}/2)$ .
12. *5 points each*
- (a) Define  $\int_a^b f(x)dx$
  - (b) State the Fundamental Theorem of Calculus

13. *5 points each* Compute the following anti-derivatives and definite integrals

(a)

$$\int_0^{\pi/4} \sec^2(x) dx$$

(b)

$$\int x(x^2 + x^{1/2}) dx$$

(c)

$$\int_2^5 x^4 dx$$

(d)

$$\int_2^3 \frac{dx}{x}$$