

Math 125 Carter Final Exam Fall 2005

Do all of your work in your blue book. Don't write on the test, I won't look at it. Write your name on only the outside of your blue book. Thank you for your hard work in this course. When making a chicken salad, try mixing the mayonaise with the vegetables before adding the chicken. If you don't eat chicken, then use tofu.

1. Compute $f'(x)$ or $\frac{dy}{dx}$ derivatives of the following (5 points each):

(a) $f(x) = x^3 - 2x^2 + 3x - 5$

(b) $y = \cos(x^2 + 1)$

(c) $y = \sqrt{(x^2 + 3x - 4)}$

(d) $f(x) = e^{\cos(x^2+1)}$

(e) $f(x) = e^{3x} \cos(2x)$

(f) $f(x) = \ln(\cosh(x))$

(g) $x^2 - y^2 = 5$

2. Compute the limit of the Newton quotient (or difference quotient) for the function (10 points):

$$f(x) = \frac{1}{(x+1)^2}$$

3. (5 points each)

- (a) Define the phrase:

$$\lim_{x \rightarrow x_0} f(x) = L.$$

- (b) State the Fundamental Theorem of Calculus.

4. Compute the following limits (5 points each):

(a) $\lim_{x \rightarrow 0} \frac{2^x - 1}{x}$

(b) $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

(c) $\lim_{x \rightarrow \infty} \frac{4x^2 - 5x + 13}{x^2 - 405x - 37}$

5. (10 points) Prove by induction,

$$1^3 + 2^3 + \dots + n^3 = \left(\frac{n(n+1)}{2} \right)^2.$$

6. (10 points) A projectile is shot upward from the edge of a 576 foot cliff and moves vertically along a straight line according to the equation,

$$s(t) = -16t^2 + 80t + 576$$

where $t \geq 0$ is measured in seconds, and the vertical position, s , is measured in meters.

- (a) Sketch a graph of the position as a function of time. Include an appropriate domain.
 - (b) When does the projectile reach its highest point?
 - (c) When does it hit the ground?
 - (d) What is the velocity of the projectile as it hits the ground ($s(t) = 0$)?
 - (e) What is the initial velocity?
7. (10 points) Sketch the graph of the function

$$f(x) = \frac{x}{x^2 + 16},$$

include critical point(s), inflection points, and any asymptotic behavior.

8. (10 points) Sketch the graph of the function

$$f(x) = (x - 2)^2(x + 1),$$

include critical point(s), inflection points, and any asymptotic behavior.

9. (10 points) Syrup flows out of a conical funnel at a rate of 3 cubic centimeters per second. The funnel is 15 cm in diameter and 20 cm in height. At what rate is the height of the funnel decreasing when the height of the syrup is 10 cm? (Hint: The volume of a cone is given by $V = \frac{\pi}{3}r^2h$.)
10. (10 points) Compute the dimensions of the rectangle of maximum area that can be inscribed in under the parabola $y = 16 - x^2$ with the base of the rectangle on the x -axis.
11. (5 points each) Compute the following anti-derivatives and definite integrals

- (a)

$$\int_1^2 x^2 dx$$

- (b)

$$\int x^{-1}(x^2 + x^{1/2})dx$$

- (c)

$$\int \frac{dx}{x}$$

- (d)

$$\int_0^\pi \sin(x)dx$$