

Diagrammatic Cohomology and Knot Invariants

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May 2008

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Plan

1. Statements
2. Type III and cocycle conditions
3. Applications of quandle cocycles
4. Other contexts
5. Infiltrations of Elaborate plans
6. Computation of some cohomology
7. Conclusion

Results

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Solutions to the YBE constructed.

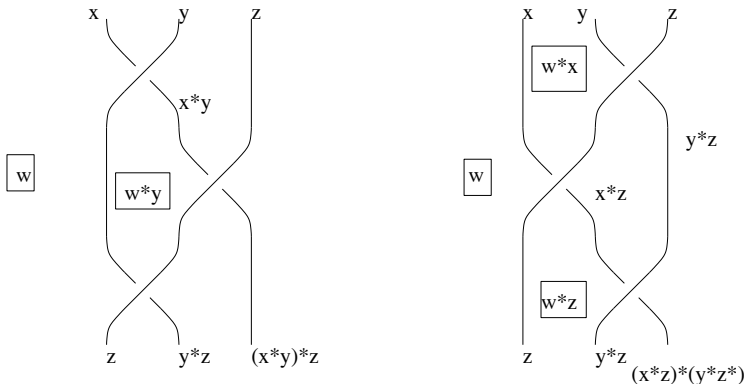
A **quandle** (Q, \triangleleft) is a set Q equipped with a binary operation

$$\triangleleft \rightarrow Q \times Q \rightarrow Q$$

that satisfies:

- (i) $x \triangleleft x = x$ for all $x \in Q$
- (ii) For all $x, y \in Q$, there exists a unique $z \in Q$ such that $z \triangleleft x = y$
- (iii) $(x \triangleleft y) \triangleleft z = (x \triangleleft z) \triangleleft (y \triangleleft z)$ for all $x, y, z \in Q$

Type III move and cocycles



$$\phi(x,y) + \phi(x^*y,z) + \phi(y,z) = \phi(y,z) + \phi(x,z) + \phi(x^*z,y^*z)$$

$$\theta(w,x,y) + \theta(w^*y,x^*y,z) + \theta(w,y,z) = \theta(w,x,z) + \theta(w^*x,y,z) + \theta(w^*z,x^*z,y^*z)$$

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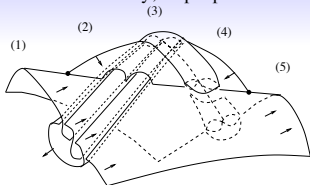
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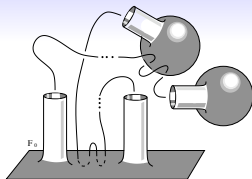
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8. applications to Lefschetz fibrations (Zablow).

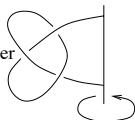
Non-invertibility, Triple point number



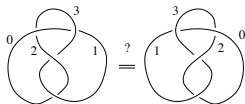
Ribbon Concordance



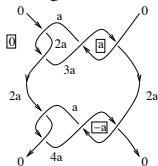
Sheet number



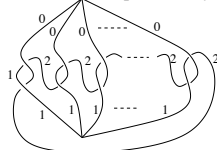
Colored chirality



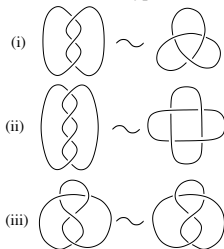
Tangle embeddings



Graph chirality



Minimal # of type III moves



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- 3-cocycles related to categorical internalization.

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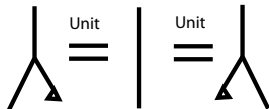
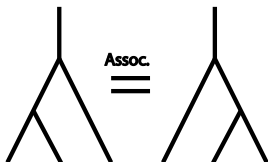
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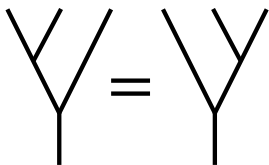
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- Enrichment of KhoHo to get non-trivial invs for surfaces;
- deeper connections between classical and quantum invariants.

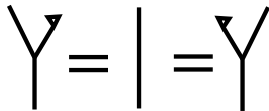
Diag Alg.



Coalgebra



Co-assoc.



Co-unit

Frobenius Algebra

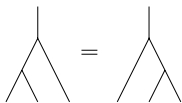
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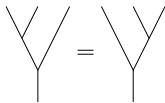
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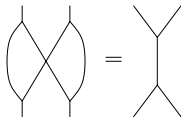
Hopf Algebra



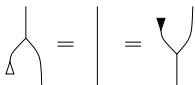
Associativity



Coassociativity

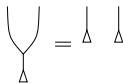


Compatibility

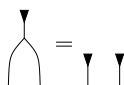


Unit

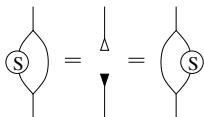
Counit



Unit is a coalgebra hom



Counit is an algebra hom



Antipode condition

Algebraic Infiltration

The cocycle cond. for assoc. algs. is given by:

$$\begin{array}{c} \begin{array}{ccc} \begin{array}{c} | \\ \diagup \quad \diagdown \\ \blacktriangle \quad | \\ | \quad \diagdown \\ \diagup \quad \diagdown \end{array} & + & \begin{array}{c} | \\ \blacktriangle \\ \diagup \quad \diagdown \\ | \quad \diagdown \\ \diagup \quad \diagdown \end{array} \end{array} \\ \\ = & \begin{array}{ccc} \begin{array}{c} | \\ \diagup \quad \diagdown \\ | \quad \diagdown \\ \blacktriangle \quad | \\ \diagup \quad \diagdown \end{array} & + & \begin{array}{c} | \\ \blacktriangle \\ \diagup \quad \diagdown \\ | \quad \diagdown \\ \diagup \quad \diagdown \end{array} \end{array} \end{array}$$

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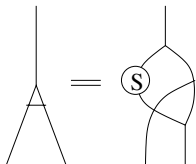
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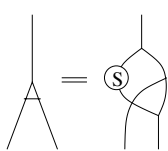
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In this case, the adjoint map is given on basis elements by $ad(a \otimes b) = b^{-1}ab$.

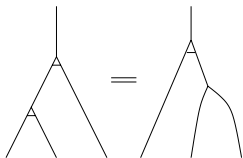
For example the adjoint map is given by:



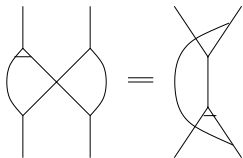
Two important properties



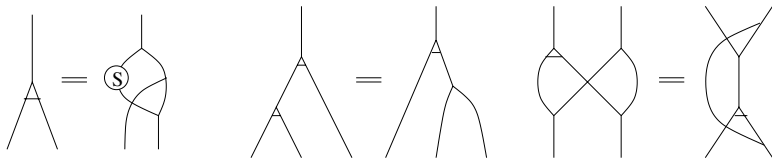
(A)



(B)

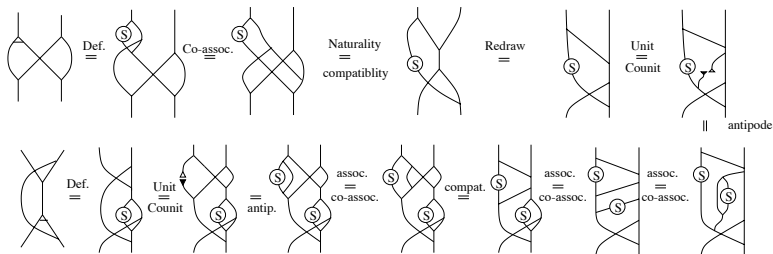


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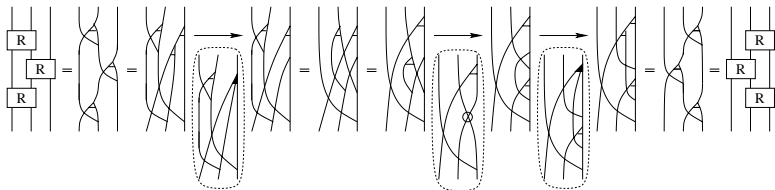


(A)

(B)



Woronowicz sol'n to YBE



$$d^{1,1}(\phi) = \text{triangle with } \phi \text{ on right} - \text{triangle with } \phi \text{ on top} + \text{triangle with } \phi \text{ on left}$$

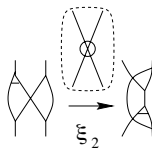
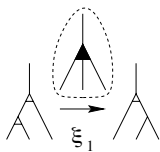
$$d^{1,1}(\circlearrowleft) = \text{triangle with arrow on right} - \text{triangle with arrow on top} + \text{triangle with arrow on left}$$

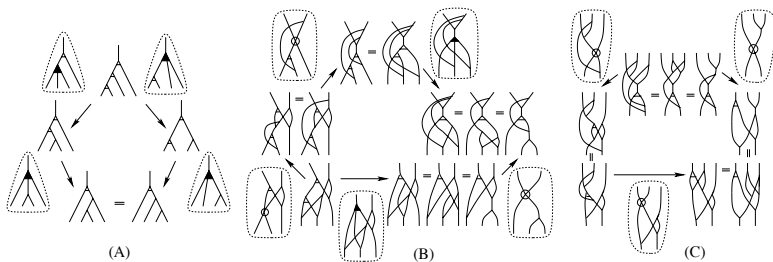
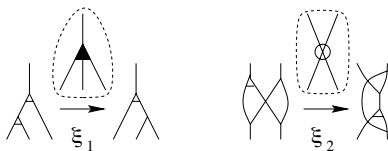
$$\eta_1 = \text{triangle with arrow on right}, \quad d^{2,1}(\text{triangle with arrow on right}) = \text{triangle with arrow on right} + \text{triangle with arrow on left} - \text{triangle with arrow on top} = 0, \quad d^{2,2}(\text{triangle with arrow on right}) = \text{two triangles meeting at a vertex} - \text{two triangles meeting at a vertex} = 0$$

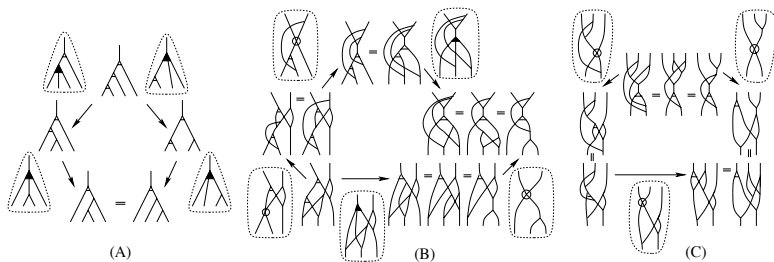
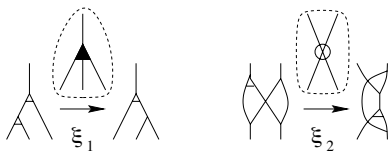
$$d^{1,1}(\phi) = \text{triangle with } \phi \text{ on bottom edge} - \text{triangle with } \phi \text{ on top edge} + \text{triangle with } \phi \text{ on left edge}$$

$$\eta_1 = \text{triangle with } \blacktriangle \text{ on bottom edge}, \quad d^{2,1}(\text{triangle with } \blacktriangle \text{ on bottom edge}) = \text{triangle with } \blacktriangle \text{ on left edge} + \text{triangle with } \blacktriangle \text{ on right edge} - \text{triangle with } \blacktriangle \text{ on top edge} = 0, \quad d^{2,2}(\text{triangle with } \blacktriangle \text{ on bottom edge}) = \text{triangle with } \blacktriangle \text{ on top edge} - \text{triangle with } \blacktriangle \text{ on bottom edge} = 0$$

$$d^{2,1}(\text{triangle with } \blacktriangle \text{ on bottom edge}) = \left(\text{triangle with } \blacktriangle \text{ on left edge} - \text{triangle with } \blacktriangle \text{ on right edge} + \text{triangle with } \blacktriangle \text{ on top edge} \right) + \left(\text{triangle with } \blacktriangle \text{ on left edge} - \text{triangle with } \blacktriangle \text{ on right edge} + \text{triangle with } \blacktriangle \text{ on top edge} \right) - \left(\text{triangle with } \blacktriangle \text{ on left edge} - \text{triangle with } \blacktriangle \text{ on right edge} + \text{triangle with } \blacktriangle \text{ on top edge} \right) = 0 \quad \text{if } \phi = \text{triangle with } \blacktriangle \text{ on left edge} + \text{triangle with } \blacktriangle \text{ on right edge}$$







$$\begin{aligned}
 d^{3,1} \left(\begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \right) &= \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \\
 &= \left(\begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \right) + \left(\begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \right) - \left(\begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \right) - \left(\begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \backslash \\ / \backslash \end{array} \right)
 \end{aligned}$$

$$d^{3,1} \left(\begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) = \begin{array}{c} | \\ \blacktriangle \\ | \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \end{array} - \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array}$$

$$= \left(\begin{array}{c} | \\ \blacktriangle \\ | \end{array} + \begin{array}{c} | \\ \blacktriangle \\ / \end{array} - \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} \right) + \left(\begin{array}{c} | \\ \blacktriangle \\ / \end{array} + \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) - \left(\begin{array}{c} | \\ \blacktriangle \\ / \end{array} + \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) - \left(\begin{array}{c} | \\ \blacktriangle \\ / \end{array} + \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right)$$

$$\begin{array}{c} | \\ \circ \\ | \end{array} + \begin{array}{c} | \\ \circ \\ / \end{array} + \begin{array}{c} | \\ \circ \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} - \begin{array}{c} | \\ \circ \\ | \end{array}$$

$$= \left(\begin{array}{c} | \\ \circ \\ | \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) + \left(\begin{array}{c} | \\ \circ \\ / \end{array} - \begin{array}{c} | \\ \blacktriangle \\ / \end{array} \right) + \left(\begin{array}{c} | \\ \circ \\ \backslash \end{array} + \begin{array}{c} | \\ \blacktriangle \\ \backslash \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) - \left(\begin{array}{c} | \\ \circ \\ | \end{array} + \begin{array}{c} | \\ \blacktriangle \\ | \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) - \left(\begin{array}{c} | \\ \circ \\ | \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right)$$

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$$\begin{array}{c} | \\ \circ \\ | \end{array} + \begin{array}{c} | \\ \circ \\ \diagdown \end{array} - \begin{array}{c} | \\ \circ \\ \diagup \end{array} = \left(\begin{array}{c} | \\ \circ \\ | \end{array} - \begin{array}{c} | \\ \blacktriangle \\ | \end{array} \right) + \left(\begin{array}{c} | \\ \blacktriangle \\ \diagdown \end{array} - \begin{array}{c} | \\ \circ \\ \diagdown \end{array} \right) - \left(\begin{array}{c} | \\ \blacktriangle \\ \diagup \end{array} - \begin{array}{c} | \\ \circ \\ \diagup \end{array} \right)$$

Example Computations

The 2-cocycle condition, the 3-cocycle condition, and the 3-coboundary condition, respectively, gives rise to the equations

$$a(x, y) + a(y^{-1}xy, z) - a(x, yz) = 0,$$

$$c(x, y, z) + c(x, yz, w) - c(y^{-1}xy, z, w) - c(x, y, zw)$$

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Theorem

For the symmetric group $G = S_3$ on three letters, we have $H_{\text{ad}}^1(kG; kG) = 0$ and $H_{\text{ad}}^2(kG; kG) \cong (kG)^{\otimes 3}$ for $k = \mathbb{C}, \mathbb{F}_2$ and \mathbb{F}_3 .

Function algs. of a finite group

G is a finite group and \mathbb{F} a field. The set \mathbb{F}^G has a Hopf algebra structure using $\mathbb{F}^{G \times G} \cong \mathbb{F}^G \otimes \mathbb{F}^G$ with comultiplication defined through $\Delta : \mathbb{F}^G \rightarrow \mathbb{F}^{G \times G}$ by $\Delta(f)(u, v) = f(uv)$ and the antipode by $S(f)(x) = f(x^{-1})$.

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Lemma

$$Z_{\text{ad}}^2(\mathbb{F}^G; \mathbb{F}^G) = 0.$$

Theorem

For any finite group G and a field \mathbb{F} , we have $H_{\text{ad}}^n(\mathbb{F}^G; \mathbb{F}^G) = 0$ for $n = 1, 2$.

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6. Details on Frob algs in Masahico's talk.
7. Details of Cat. SD in Alissa's talk.

Dankeschöne!