

Math 115 Carter Sample Final Exam

General Instructions: Write your name on only the outside of your blue book. Put your test paper inside your blue book as you leave. Do all of your work and write your solutions inside your blue book. Do not write on this test sheet. Solve each of the following problems.

1. Solve the inequalities (*5 points each*):

(a) $0 \leq x - 4$

(b) $10 < |4x - 6|$

2. Determine the domains of the following functions (*5 points each*):

(a) $f(x) = \sqrt{x - 4}$

(b) $f(x) = \frac{x-7}{x-5}$

3. Compute the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the function $f(x) = \frac{1}{x+1}$. (*10 points*)

4. Compute $f(g(x))$ and $g(f(x))$ for the function $y = f(x)$ and $y = g(x)$ that appears below. BE SURE TO LET ME KNOW WHICH IS WHICH! (*5 points each*)

(a) $f(x) = x^2$; $g(x) = 3x + 2$

(b) $f(x) = \sqrt{x}$; $g(x) = x - 4$

(c) $f(x) = \frac{1}{x}$; $g(x) = x + 3$

5. Sketch the graph of each of the following functions (*5 points each*):

(a) $f(x) = 3x^2 + 2$

(b) $f(x) = 2x^2 + 50x - 12$

(c) $f(x) = 3|x + 5| - 27$

(d) $f(x) = \frac{1}{x} - 2$

6. Determine the equations and sketch the graphs of the following lines (*5 points each*):

(a) The line that passes through the point $(0, -9)$ and that has slope 3.

(b) The line whose x -intercept is $(-180, 0)$ and whose y -intercept is $(0, 120)$.

(c) The line that passes through the point $(6, 5)$ and that is parallel to the line $y = 5x - 4$.

7. Determine the maximum value for the product $A = xy$ when $2x + 3y = 500$.

8. Sketch the graphs of the following functions. Make sure that you indicate horizontal and vertical scales clearly. (*10 points each*)

(a) $y = x(x - 10)$

(b) $y = 10x^2 - 40x - 76$

(c) $y = x + 1/x$

(d) $y = (x - 1)(x + 2)(x - 4)(x + 1)^2$

(e) $y = \frac{(x-2)}{(x-1)(x+4)}$

(f) $y = 3e^{(2x-4)}$

(g) $y = 1 - e^{-0.06x}$

(h) $y = 10 \cos(3x - \pi)$

(i) $y = \tan \pi x$

(j) $y = 4 \sin(\pi x) + 2$

(k) $y = 10 \sec(x - \pi/2)$

9. (*10 point*) A bond will pay \$10,000 in 10 years. If the current interest rate is 6% per year continuously compounded interest, then what would you be willing to pay?

10. Solve the following inequality: (10 points)

$$\frac{x^2}{x^2 - 4} < 0.$$

11. Verify the following identities: (10 points each)

(a) $\tan(x) + \sec x = \frac{\cos(x)}{1 - \sin(x)}$

(b) $\frac{\sin(x)}{1 + \cos(x)} = \frac{1 - \cos(x)}{\sin(x)}$

(c) $\sec(x) = \sin(x)(\tan(x) + \cot(x))$

(d) $(\tan(x) - \sec(x))^2 = \frac{1 - \sin(x)}{1 + \sin(x)}$

12. Solve for all values of $x \in [0, 2\pi]$ (10 points):

$$4 \sin^2(2x) = 1.$$

(a) Solve the following equations. Check for extraneous solutions (10 points each).

i.

$$\frac{100}{2 + e^x} = 25$$

ii.

$$\log(x + 4) - \log(2x - 3) = \log(2)$$

iii.

$$\ln(x) + \ln(x + 3) = 2$$

(b) Simplify $\sin(A + B) - \sin(A - B)$.

(c) Solve the following triangles (in each of them (α, β, γ) are the respective angles opposite to the sides (a, b, c)) (10 points each):

i. $a = 5, b = 12, c = 13$

ii. $\gamma = 90^\circ, c = 11, b = 8$.

iii. $a = 14, b = 12, \gamma = 42^\circ$.

(d) Evaluate $\arcsin(\cos(\frac{5\pi}{6}))$

(e) Use the graphing capabilities of your calculator to approximate the solutions (that are in the interval $[0, 2\pi]$) to the equation:

$$x \cos x - 2 = 0$$

(f) Find all solutions for $0 \leq x < 2\pi$,

$$3 \sin^2 x = 3 \sin x + 2$$

(g) Solve the triangle, if possible: $A = 131^\circ$; $C = 23^\circ$; $b = 10$

(h) Solve the triangle, if possible: $C = 28^\circ 43'$; $a = 6M$; $b = 9M$