General Information. This study guide consists of three parts. The first part is a compendium of definitions and proofs that you should know. I will ask 3 or 4 questions about this material. In the case of the definitions, I have written the definition as I expect you to reproduce it. Memorize these definitions and proofs and you will secure these points. The second part is a detailed list of problems, segregated by topic, that I will use to model the test questions. The third part consists of copies of the last two finals that I have given.

According to my reading of http://www.southalabama.edu/registrar/dates.htm the final is scheduled for Saturday, December 11, 2010 from 10:30 a.m. - 12:30 p.m.

Required Definitions and Proofs

Definitions

1. A function is continuous at the point \( x = c \) if and only if

\[
\lim_{x \to c} f(x) = f(c).
\]

That is the function is defined at \( x = c \), the limit exists at \( x = c \) and the limit is equal to the function’s value at that point.

2. We say that

\[
\lim_{x \to c} f(x) = L
\]

if for any \( \epsilon > 0 \), there is a \( \delta > 0 \) such that \( |f(x) - L| < \epsilon \) provided that \( 0 < |x - c| < \delta \).

See http://www.youtube.com/watch?v=wSTJLUAu5ZI

3. A function \( y = f(x) \) is differentiable at \( x = c \) provided the limit

\[
\lim_{h \to 0} \frac{f(c + h) - f(c)}{h}
\]

exists. See http://www.youtube.com/watch?v=Ldi6calfjNA
Proofs

1. Be able to prove the product rule:

\[(fg)'(x) = f(x)g'(x) + f'(x)g(x).\]

See

http://www.youtube.com/watch?v=RALGAHo9L14

2. Be able to prove

\[\lim_{x \to 0^+} \frac{\sin(x)}{x} = 1\]

See

http://www.youtube.com/watch?v=o6S6RbfhRTU

3. Be able to prove that

\[\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0\]

See

http://www.youtube.com/watch?v=encQhh2JeYc

4. Be able to prove that

(a) \[\sum_{j=1}^{n} j = \frac{n(n+1)}{2}\]

(b) \[\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}\]

(c) \[\sum_{j=1}^{n} j^3 = \left(\frac{n(n+1)}{2}\right)^2\]

See

http://www.youtube.com/watch?v=-usK5CMpUPo
1 Problems

1.1 Computing Limits

Pay particular attention to complicated problems that involve combinations of chain rule, trig functions, inverse trig functions, product and/or quotient rules, and logs and exponentials.

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1.2 Computing Derivatives

Be able to sketch the graph of each of the functions indicated below and similar functions.

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1.3 Applications

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Be able to sketch the graph of each of the functions indicated below and similar functions.
1.4 Anti-derivatives and integrals

2 Old Finals

Math 125  Carter  Final Exam  Fall 2009

General Instructions. Do all of your work in your blue book. Don’t write on the test, I won’t look at it. Write your name on only the outside of your blue book. Thank you for your hard work in this course.

Garlic-lemon-butter is nice for a sautéed shrimp or fish fillet. Melt a stick of butter along with an ounce of lemon juice. Crush 2 to 3 large cloves of garlic and slice thinly. Melt butter, lemon juice, and garlic over low heat being careful not to scald the butter. In a shallow skillet add a fish fillet (snapper or tapia) or jumbo shrimp that have been de-headed, de-viened, split, and pulled from the shells. Cook over low heat until the seafood is done. Add capers to taste.

1. Compute $f'(x)$ or $\frac{dy}{dx}$ derivatives of the following (5 points each):

(a) $f(x) = 2x^3 - 4x^2 + 3x - 14$

(b) $y = \sin(x^3)$

(c) $y = \sqrt{x^2 + 1}$
2. Use the definition of the derivative to prove the product rule (5 points):
\[
\frac{d(f \cdot g)}{dx} = f(x) \frac{dg}{dx} + \frac{df}{dx} g(x).
\]

3. (5 points) Define the phrase:
\[
\lim_{x \to x_0} f(x) = L.
\]

4. Compute the following limits (5 points each):
   (a) \[
   \lim_{x \to 0} \frac{\cos(x) - 1}{x}
   \]
   (b) \[
   \lim_{N \to \infty} \frac{N(N + 1)(2N + 1)}{6N^3}
   \]

5. (10 points) Prove by induction,
\[
1 + 2 + \cdots + n = \left( \frac{n(n + 1)}{2} \right).
\]

6. (10 points) Use the power rule to compute the equation of the line tangent to the curve \(y = x^3\) at the point \(x = 2\).
7. (10 points) Sand falls in a conical pile at the rate of 5m$^3$ per minute. The radius is always 3 times as large as the height. At what rate is the height changing when the height is 4 meters?

8. (10 points) The volume of a sphere is increasing at the rate of 2 cubic centimeters per second. How fast is the radius increasing when the radius is 20 centimeters?

9. (10 points) Classify the critical point $c = \frac{\pi}{4}$ of the function $y = \sin(x) \cos(x)$ as a local maximum, local minimum, or neither. Justify your answer with either a graph, using the 1st derivative test, or the second derivative test.

10. (10 points each) Sketch the graph of the following functions:

   (a) 
   \[ f(x) = x^3 - x \]

   (b) 
   \[ f(x) = \sin(2x) \text{ for } x \in [0, 2\pi]. \]

   (c) 
   \[ f(x) = \frac{1}{x^2 - 4x + 3} \]

11. (5 points each) Compute the following anti-derivatives and definite integrals

   (a) 
   \[ \int_{2}^{3} x^2 \, dx \]
Math 125  Carter  Final Exam  Spring 2010

**General Instructions.** Do all of your work in your blue book. Don’t write on the test: I won’t look at it. Write your name **on only** the outside of your blue book. Please insert your test paper into your blue book as you leave. Thank you for your hard work in this course. This test is three pages.

A steamy bowl of oatmeal provides a filling and nutritious breakfast. Slice a banana, add a pinch of salt, a half a cup of oats, and a cup of milk. Stir frequently over medium to high heat. Add raisins, brown sugar, cinnamon, and cold milk to taste.

1. Compute \( f'(x) \) or \( \frac{du}{dx} \) derivatives of the following (5 points each):

   (a) \( f(x) = 3x^2 - 4x - 9 \)
   
   (b) \( y = e^{(x^3)} \)
   
   (c) \( y = \frac{x}{\sqrt{x^2 + 1}} \)
   
   (d) \( f(x) = x^2 \tan(x) \)
   
   (e) \( f(x) = \ln((x^2 - 1)(x + 2)) \)
   
   (f) \( f(x) = \int_0^x \sin(-t^2) \, dt \)
(g) \[ y^2 - x^2 = 16 \]

2. Use the definition of the derivative to prove the product rule (5 points):

\[
\frac{d(f \cdot g)}{dx} = f(x) \frac{dg}{dx} + \frac{df}{dx} g(x).
\]

3. (5 points) Define the phrase:

\[
\lim_{x \to x_0} f(x) = L.
\]

4. Compute the following limits (5 points each):

(a) \[
\lim_{x \to 0} \frac{\sin(3x)}{\sin(4x)}
\]

(b) \[
\lim_{N \to \infty} \frac{N(N + 1)}{2N^2}
\]

5. (10 points) An open box is to be made from a 24cm-by-24cm piece of cardboard by cutting out four corners and folding up the sides. What are the dimensions of the box of maximum volume that can be made in this way?

6. (10 points) Use the power rule to compute the equation of the line tangent to the curve \( y = x^2 \) at the point \( x = 4 \).

7. (10 points) Coffee drips at a rate of 2 cubic centimeters per minute from a 14 centimeter tall Melita cone whose radius is 7 centimeters. How fast is the height of the coffee decreasing when the height is 5 centimeters? The volume of a cone is \( V = \frac{\pi}{3} r^2 h \).
8. (10 points) Sketch the graph of the function \( y = \sin(x) \cos(x) \) over the interval \([-2\pi, 2\pi]\). Indicate all of the critical points and the inflection points.

9. (10 points each) Sketch the graph of the following functions:

(a) 
\[ f(x) = x^4 - x^2 \]

(b) 
\[ f(x) = (x + 3)(x + 1)(x - 2) \]

(c) 
\[ f(x) = \frac{x - 1}{x^2 + x} \]

10. (5 points each) Compute the following anti-derivatives and definite integrals

(a) 
\[ \int_{1}^{4} (2x + 5) \, dx \]

(b) 
\[ \int \frac{x^2 + x}{x^{1/2}} \, dx \]

(c) 
\[ \int e^{2x} \, dx \]
11. (10 points) One barrel of oil is roughly 160 liters. The oil well 300 miles due south of Mobile is leaking at a rate of 5000 barrels per day. I am writing this problem on Thursday. The oil slick is 80 miles from east to west and 30 miles north to south. The source of the oil is at the southwest corner of the slick.

In this item, I want you to formulate the question. Let me know the problems that you would formulate, which units need to be converted, the items that are not given that you should know in order to solve the problem, and how you would approach the problem using things that you have learned this semester. Note: I am not asking you solve the problem you pose, but I am asking you to formulate a given real world scenario as a calculus problem.