General Instructions. Do all of your work in your blue book. Don’t write on the test, I won’t look at it. Write your name on only the outside of your blue book. Thank you for your hard work in this course.

Garlic-lemon-butter is nice for a sautéed shrimp or fish fillet. Melt a stick of butter along with an ounce of lemon juice. Crush 2 to 3 large cloves of garlic and slice thinly. Melt butter, lemon juice, and garlic over low heat being careful not to scald the butter. In a shallow skillet add a fish fillet (snapper or tapia) or jumbo shrimp that have been de-headed, de-viened, split, and pulled from the shells. Cook over low heat until the seafood is done. Add capers to taste.

1. Compute \( f'(x) \) or \( \frac{dy}{dx} \) derivatives of the following (5 points each):

   (a) \[ f(x) = 2x^3 - 4x^2 + 3x - 14 \]

   (b) \[ y = \sin (x^3) \]

   (c) \[ y = \sqrt{x^2 + 1} \]

   (d) \[ f(x) = e^{x^2 + \tan(x)} \]

   (e) \[ f(x) = \sinh (x) \ln (x^2 + x + 1) \]

   (f) \[ f(x) = \int_0^x e^{-t^2} \, dt \]

   (g) \[ x^2 + y^2 = 16 \]

2. Use the definition of the derivative to prove the product rule (5 points):

   \[
   \frac{d(f \cdot g)}{dx} = f(x) \frac{dg}{dx} + \frac{df}{dx} g(x).
   \]

3. (5 points) Define the phrase:

   \[
   \lim_{x \to x_0} f(x) = L.
   \]
4. Compute the following limits (5 points each):

(a) \[ \lim_{x \to 0} \frac{\cos (x) - 1}{x} \]

(b) \[ \lim_{N \to \infty} \frac{N(N + 1)(2N + 1)}{6N^3} \]

5. (10 points) Prove by induction,

\[ 1 + 2 + \cdots + n = \left(\frac{n(n + 1)}{2}\right). \]

6. (10 points) Use the power rule to compute the equation of the line tangent to the curve \( y = x^3 \) at the point \( x = 2 \).

7. (10 points) Sand falls in a conical pile at the rate of \( 5 \text{m}^3 \) per minute. The radius is always 3 times as large as the height. At what rate is the height changing when the height is 4 meters?

8. (10 points) The volume of a sphere is increasing at the rate of 2 cubic centimeters per second. How fast is the radius increasing when the radius is 20 centimeters?

9. (10 points) Classify the critical point \( c = \frac{\pi}{4} \) of the function \( y = \sin (x) \cos (x) \) as a local maximum, local minimum, or neither. Justify your answer with either a graph, using the 1st derivative test, or the second derivate test.

10. (10 points each) Sketch the graph of the following functions:

(a) \[ f(x) = x^3 - x \]

(b) \[ f(x) = \sin (2x) \quad \text{for} \quad x \in [0, 2\pi]. \]
(c) \[ f(x) = \frac{1}{x^2 - 4x + 3} \]

11. (5 points each) Compute the following anti-derivatives and definite integrals

(a) \[ \int_2^3 x^2 \, dx \]

(b) \[ \int (x^2 + x)x^{-1/2} \, dx \]

(c) \[ \int \frac{dx}{x} \]

(d) \[ \int_{-\pi/2}^{\pi/2} \sin(x) \, dx \]