1. Define the italicized terms (5 points each):

   (a) The set \( \{A_1, A_2, \ldots, A_k\} \) is linearly independent

   (b) The set \( \{A_1, A_2, \ldots, A_k\} \) spans a vector space \( V \)

   (c) The null space of an \((m \times n)\)-matrix

   (d) The span of a set of vectors \( \{A_1, A_2, \ldots, A_k\} \)

2. (10 points) The reduced row echelon form of the matrix \( A \) that is associated to the homogeneous system of equations

\[
\begin{align*}
5x + 4y + 3z + 6w &= 0 \\
x + 3y + 2z + 2w &= 0 \\
3x - 2y - z + 2w &= 0
\end{align*}
\]

is

\[
\begin{bmatrix}
1 & 0 & \frac{1}{11} & \frac{10}{11} \\
0 & 1 & \frac{7}{11} & \frac{4}{11} \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

   (a) Determine the solution set.

   (b) Give a spanning set for the column space.

3. Solve the system of equations (10 points)

\[
\begin{align*}
x + 2y &= 3 \\
4x + 8y + 2z &= 14 \\
x + 2y + z &= 4
\end{align*}
\]

4. (10 points) Compute the matrix product:

\[
\begin{bmatrix}
1/5 & 4/5 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
5 & -4 & 20 \\
0 & 1 & 0
\end{bmatrix}
\]
5. (a) (5 points) Write the augmented matrix for the system of equations:

\[
\begin{align*}
  x + y &= 1 \\
 2x - y &= 2
\end{align*}
\]

(b) (15 points) By successively performing the corresponding row operations to the
(2 \times 2)-identity matrix, \( I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), solve the system of equations.

6. (10 points) Give an example of a linearly dependent set of 3 vectors in \( \text{M}(2,1) \) such
that any two vectors taken from the set form a linearly independent set.

7. (10 points) Give an equation of the form \( Ax + By + Cz = 0 \) such that the vectors
\( X = [1, 2, 1]^t \) and \( Y = [1, 0, 3]^t \) satisfy the equation.

8. (10 points) Is the matrix

\[
A = \begin{bmatrix}
  0 & 1 & 2 & 2 & 4 \\
  0 & 0 & 1 & 2 & 4 \\
  0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

in reduced row echelon form? If not, then row reduce it. Indicate your steps clearly.