Water runs into a conical tank at the rate of 9 ft³/min. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep. The volume of a cone is given by the formula $V = \frac{\pi}{3}r^2h$ where $r$ denotes the radius of the cone, $h$ denotes the height, and $V$ denotes the volume.

Solution.

- Let $V$ denote the volume of the water in the tank.
- Let $h$ denote the height of the water in the tank.
- Let $r$ denote the radius of the water in the tank.
- We are given that $\frac{dV}{dt} = 9$ ft³/min.
- $V = \frac{\pi}{3}r^2h$.
- Since corresponding parts of similar triangles are proportionate, $\frac{r}{h} = \frac{5}{10} = \frac{1}{2}$.

We substitute, $r = h/2$ into the formula for volume and obtain

$$V = \frac{\pi}{3} \left( \frac{h}{2} \right)^2 h = \frac{\pi h^3}{4 \cdot 3}.$$

Differentiate both sides with respect to time (and apply the chain rule):

$$\frac{dV}{dt} = \frac{3\pi h^2}{4 \cdot 3} \frac{dh}{dt}.$$

Solve for $\frac{dh}{dt}$:

$$\frac{dh}{dt} = \left( \frac{4}{3} \frac{dV}{dt} \right) / (\pi h^2).$$

Substitute the rate of change of the volume and the instant in question:

$$\frac{dh}{dt} \bigg|_{h=5} = \left( \frac{4 \cdot 9}{(\pi \cdot 6^2)} \right) = \frac{1}{\pi}. $$