Name ________________________________

For each of the functions given below: (1) evaluate the function at the end-points of the interval, (2) determine each of the critical points, (3) evaluate the function at the critical points, (4) compare the $y$-values that you have found, and (4) state something to the effect that, “$y$ achieves a minimal value of $y = y_m$ when $x = x_m$. Meanwhile, $y$ achieves a Maximal value of $y = y_M$ when $x = x_M$. Of course, it is up to you to determine $x_m, x_M, y_m$, and $y_M$.

1. \[ f(x) = 4 - x^2, \quad -3 \leq x \leq 1. \]

Solution.

- $f(-3) = 4 - (-3)^2 = 4 - 9 = -5$.
- $f(1) = 4 - (1)^2 = 3$.
- $f'(x) = -2x$.
- Critical points: $-2x = 0 \quad x = 0$.
- At $x = 0$, the function’s value is $f(0) = 4$.

The function $y = 4 - x^2$ defined on $[-3, 1]$ achieves a Maximal value of $y = 4$ at $x = 0$. The function $y = 4 - x^2$ defined on $[-3, 1]$ achieves a minimal value of $y = -5$ at $x = -3$.

2. \[ g(x) = \sqrt{4 - x^2}, \quad -2 \leq x \leq 1. \]

Solution.

- $g(-2) = \sqrt{4 - (-2)^2} = 0$
- $g(1) = \sqrt{4 - (1)^2} = \sqrt{3}$.
- $g'(x) = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$.
- Critical points: $x = 0$ and $x = \pm 2$. Only $x = 0$ is relevant to the question since $x = 2$ outside the domain, and we have evaluated $g(-2)$ earlier.

$g(0) = 2$.

The function $y = \sqrt{4 - x^2}$ defined on $[-3, 1]$ achieves a Maximal value of $y = 2$ at $x = 0$. The function $y = \sqrt{4 - x^2}$ defined on $[-3, 1]$ achieves a minimal value of $y = 0$ at $x = -2$. 