1. State the definition of a relation between sets $X$ and $Y$.

2. State the definition of a function between sets $X$ and $Y$.

3. Prove by induction:

   $$1 + 2 + \cdots + n = \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

4. Prove by induction:

   $$1^2 + 2^2 + \cdots + n^2 = \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
5. Prove by induction:

\[1^3 + 2^3 + \cdots + n^3 = \sum_{k=1}^{n} k^3 = \left[ \frac{n(n + 1)}{2} \right]^2\]