Math 227 Carter Sample 1 Spring 2007

Sample Instructions. Do all your work in your blue books. Write your solutions in your blue book. Show all work. Write your name on only the outside of your blue book. Write neatly, and use complete sentences when appropriate. My hope is that you do well on this exam. Good luck.

1. Among the three points $(2, 3, 4)$, $(-2, 8, 7)$, and $(2, -3, -3)$ which is the closest to the $(xy)$-plane?

2. Compute the equation of the sphere that has $(1, 3, 7)$ as its center and that has radius $8$.

3. Give the equation of the line in parametric form that passes through $P = (2, 3, 4)$ and $Q = (-2, 8, 7)$.

4. Give the equation of the plane that passes through $P = (2, 3, 4)$, $Q = (-2, 3, 1)$, $R = (1, 2, 4)$. What is the area of the triangle that has these points as its vertices?

5. (10 points) Compute the equation of the line of intersection between the planes $x + 2y + 3z = 6$ and $y + z = 1$.

6. (10 points) Find the equation of the plane that is perpendicular to the vector $(1, 1, 1)$, and that passes through the point $(-2, 3, 4)$.

7. (20 points) Match the figure with the equation:

   (a) $z = x^2 + y^2$

   (b) $z = \sqrt{x^2 + y^2}$

   (c) $z = 1 + x^2 - y^2$. 

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8. For the parametric curve \( \vec{r}(t) = 3 \cos(t) \hat{i} + 4 \sin(t) \hat{j} \) Determine \( \vec{r}'(t) \) and compute \( \vec{r}(t) \cdot \vec{r}'(t) \).

9. Sketch the level curves to the surface \( z = \sqrt{9 - x^2 - y^2} \), for the values \( z = 9, \ z = 8, \ z = 5, \) and \( z = 0 \). Describe the surface.

10. (15 points) Sketch the level curves \( z = -1, \ z = 0, \) and \( z = -1 \) to the surface \( z = y - \sin(x) \). Give a description of the surface in realistic terms.