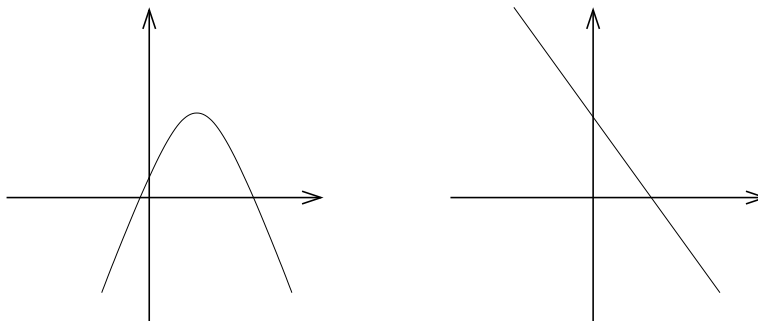


Math 125 Carter Study Guide for Final Fall 2005

Do all of your work in your blue book. Don't write on the test, I won't look at it. Write your name on only the outside of your blue book. Thank you for your hard work in this course. Do well! Buy low, sell high.

1. *5 points* In the Figure below one of the graphs represents a function; the other represents its derivative. Which is which and why?



2. Compute $f'(x)$ or $\frac{dy}{dx}$ derivatives of the following (*5 points each*):

(a) $f(x) = -x^2 + 4x + 5$

(b) $y = \sin(2x)$

(c) $y = \sqrt{(x^3 + 3x - 4)}$

(d) $f(x) = e^{x^2+1}$

(e) $f(x) = \ln(\cos(x))$

(f) $xy - y^3 = 5$

3. Compute the limit of the Newton quotient for the function *5 points*:

$$f(x) = \sqrt{x - 2}$$

4. Compute the following limits *5 points*:

(a) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

(b) $\lim_{x \rightarrow 9} \frac{x^2 - 81}{x - 9}$

(c) $\lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 - 5x + 13}{34x^4 + 18x^3 - 2x^2 - 405x - 37}$

5. *5 points* Prove by induction,

$$1^2 + 2^2 + \dots + N^2 = \frac{(N)(N + 1)(2N + 1)}{6}.$$

6. *10 points* A projectile is shot upward from the edge of a 20 meter cliff and moves vertically along a straight line according to the equation,

$$s(t) = -5t^2 + 45t + 20$$

where $t \geq 0$ is measured in seconds, and the vertical position, s , is measured in meters.

- (a) Sketch a graph of the position as a function of time. Include an appropriate domain.
- (b) When does the projectile reach its highest point?
- (c) What is the velocity of the projectile as it hits the ground ($s(t) = 0$)?
- (d) When does the projectile pass the edge of the cliff?

7. *10 points* Sketch the graph of the function

$$f(x) = \frac{1}{x^2 + 1},$$

include critical point(s), inflection points, and any asymptotic behavior.

8. *10 points* Sand falls in a conical pile at a constant rate of 3 cubic meters per second. The radius of the cone is always twice the height. How fast is the height increasing when the height is 4 meters? (Hint: The volume of a cone is given by $V = \frac{\pi}{3}r^2h$)
9. *10 points* Show that among all the rectangles that are inscribed in a circle of radius 1 the rectangle that has the maximum area is a square. Hint: Use a cartesian coordinate system.
10. *10 points* Use your calculator to ESTIMATE

$$\int_1^2 \ln x \, dx$$

By subdividing the interval $[0, 1]$ into 5 subintervals of length $1/5$.

11. *10 points* Compute the equation of the line tangent to the curve, $y = \cos(x)$ at the point $(\pi/6, \sqrt{3}/2)$.
12. *5 points each*
- (a) Define $\int_a^b f(x)dx$
 - (b) State the Fundamental Theorem of Calculus

13. *5 points each* Compute the following anti-derivatives and definite integrals

(a)

$$\int_0^{\pi/4} \sec^2(x) dx$$

(b)

$$\int x(x^2 + x^{1/2}) dx$$

(c)

$$\int_2^5 x^4 dx$$

(d)

$$\int_2^3 \frac{dx}{x}$$