1. (8 points each) Compute the following:

(a) \[ \int x \ln(x) \, dx \]

(b) \[ \int \frac{1}{1 + x^2} \, dx \]

(c) \[ \int e^x \sin(x) \, dx \]

(d) \[ \int \sin^3(x) \, dx \]

(e) \[ \int \frac{1}{x^2 + 3x + 2} \, dx \]

(f) \[ \int_1^\infty \frac{dx}{x} \]

2. (7 points) Define \[ \lim_{n \to \infty} a_n = L. \]

3. (10 points) Compute the arc length \( \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx \) of \( f(x) = x^{3/2} \) for \( x \in [1, 2] \).

4. (10 points) Compute the volume of the solid obtained by rotating the region bounded by \( y = x^2 \) and \( y = x \) about the \( y \)-axis.
5. (10 points) Compute the centroid of the region that is bounded by $y = x^2$ and $y = \sqrt{x}$.

Note that if $f_1(x) \leq f_2(x)$, then

$$x_{\text{CM}} = \frac{\rho \int_a^b x(f_2(x) - f_1(x)) \, dx}{\rho \int_a^b (f_2(x) - f_1(x)) \, dx}$$

and

$$y_{\text{CM}} = \frac{1}{2} \frac{\rho \int_a^b ((f_2(x))^2 - (f_1(x))^2) \, dx}{\rho \int_a^b (f_2(x) - f_1(x)) \, dx}$$

where $\rho$ indicates the density measured in mass per unit area. An alternate formula for $y_{\text{CM}}$ is

$$x_{\text{CM}} = \frac{\rho \int_a^b y(g_2(y) - g_1(y)) \, dy}{\rho \int_a^b (g_2(y) - g_1(y)) \, dy}$$

Where the region is bounded by $x = g_1(y)$ and $x = g_2(y)$ for $g_1(x) \leq g_2(x)$.

6. (10 points) Compute the (Taylor) MacLaurin polynomial $T_5(x) = \sum_{j=0}^{5} \frac{f^{(j)}(a)}{j!} (x - a)^j$ of the indicated function at the point $a$.

$$f(x) = e^x; a = 0.$$
General Instructions: In this document you will find 4 separate sample tests. The actual test will be similar in format to any one of these tests. I anticipate making the problem difficulty level about the same as it is here. But I cannot always guarantee the difficulty level. I strongly suggest that you make yourself a 5th test of the same format; include problems at this level of difficulty and a few problems that are more difficult.

1. (8 points each) Compute the following:
   (a) \[ \int xe^{4x} \, dx \]
   (b) \[ \int \sqrt{4 - x^2} \, dx \]
   (c) \[ \int e^{2x} \cos(x) \, dx \]
   (d) \[ \int \cos^3(x) \, dx \]
   (e) \[ \int \frac{1}{x^2 - 4x + 3} \, dx \]
   (f) \[ \int_0^1 \frac{dx}{\sqrt{x}} \]

2. (7 points) Define \[ \lim_{n \to \infty} a_n = L. \]

3. (10 points) Compute the arc length \( \int_a^b \sqrt{1 + (f'(x))^2} \, dx \) of \( f(x) = \ln \cos(x) \) for \( x \in [0, \pi/4] \).

4. (10 points) Compute the volume of the solid obtained by rotating the region bounded by \( y = x^2 \) and \( y = \sqrt{x} \) about the y-axis.

5. (10 points) Assuming a spring constant of \( k = 400 \text{ kg/sec}^2 \), compute the work in Joules (kilogram meter-squared per second-squared) that is required to stretch the spring 10 centimeters beyond equilibrium.
6. (10 points) Compute the (Taylor) MacLaurin polynomial \( T_5(x) = \sum_{j=0}^{5} \frac{f^{(j)}(a)}{j!} (x - a)^j \) of the indicated function at the point \( a \).

\[
f(x) = \sin(x); \ a = 0.
\]
Math 126 Carter Sample Test 2 (version c) Spring 2011

**General Instructions:** In this document you will find 4 separate sample tests. The actual test will be similar in format to any one of these tests. I anticipate making the problem difficulty level about the same as it is here. But I cannot always guaranty the difficulty level. I strongly suggest that you make yourself a 5th test of the same format; include problems at this level of difficulty and a few problems that are more difficult.

1. (8 points each) Compute the following:
   
   (a) \[ \int x \cos(2x) \, dx \]
   (b) \[ \int t \sqrt{1 - t^2} \, dt \]
   (c) \[ \int \sec^3(x) \, dx \]
   (d) \[ \int \sec^2(x) \, dx \]
   (e) \[ \int \frac{1}{x^2 + x + 1} \, dx \]
   (f) \[ \int_{1}^{\infty} \frac{dx}{x^{3/2}} \]

2. (7 points) Define \[ \lim_{n \to \infty} a_n = L. \]

3. (10 points) Compute the arc length \( \int_{a}^{b} \sqrt{1 + (f'(x))^2} \, dx \) of \( f(x) = 9 - 3x \) for \( x \in [1, 3] \).

4. (10 points) Compute the volume of the solid obtained by rotating the region bounded by 
\( y = x^2 \) and \( y = \sqrt{x} \) about the \( x \)-axis.

5. (10 points) Compute the work needed in building a brick (density 80 pounds per cubic-foot) structure that is a tower of height 20 feet and square base of side 10 feet.

6. (10 points) Compute the (Taylor) MacLaurin polynomial \( T_5(x) = \sum_{j=0}^{5} \frac{f^{(j)}(a)}{j!}(x - a)^j \) of the indicated function at the point \( a \).
   
   \[ f(x) = \cos(x); \ a = 0. \]
General Instructions: In this document you will find 4 separate sample tests. The actual test will be similar in format to any one of these tests. I anticipate making the problem difficulty level about the same as it is here. But I cannot always guarantee the difficulty level. I strongly suggest that you make yourself a 5th test of the same format; include problems at this level of difficulty and a few problems that are more difficult.

1. (8 points each) Compute the following:
   (a) \( \int x \sin(3x) \, dx \)
   (b) \( \int \frac{1}{\sqrt{1 - x^2}} \, dx \)
   (c) \( \int e^x \sin(3x) \, dx \)
   (d) \( \int \tan^2(x) \, dx \)
   (e) \( \int \frac{1}{x(x-1)^2} \, dx \)
   (f) \( \int_1^\infty x^{-21/20} \, dx \)

2. (7 points) Define \( \lim_{n \to \infty} a_n = L \).

3. (10 points) Compute the arc length \( \int_a^b \sqrt{1 + (f'(x))^2} \, dx \) of \( f(x) = 3x + 1 \) for \( x \in [0, 3] \).

4. (10 points) Compute the volume of the solid obtained by rotating the region bounded by \( y = x^2 \) and \( y = x \) about the \( y \)-axis. \( y = x^2 \) and \( y = x \) about the \( x \)-axis.

5. (10 points) Water has a density of \( 10^4 \) kilograms per cubic meter. Compute the force against a metal plate that is submerged in water and that is the shape of an isosceles triangle with base 1 meters and height 2 meter. The vertex is at the water level.

6. (10 points) Compute the (Taylor) MacLaurin polynomial \( T_5(x) = \sum_{j=0}^{5} \frac{f^{(j)}(a)}{j!} (x - a)^j \) of the indicated function at the point \( a \).

\( f(x) = \ln(x + 1); a = 0 \).