Basic Definitions 20 points

Be able to state precisely the definitions of the terms specified below. The terms to be defined appear in boldface. The definitions that I expect you to reproduce appear beside the terms.

1. **Subspace of a Vector Space**. A subset $W \subset V$ of a vector space, $V$, is said to be a **subspace** if
   - $0 \in W$
   - $\alpha v + \beta w \in W$ whenever $\alpha, \beta \in \mathbb{R}$ and $v, w \in W$.

2. **The Span of a set of vectors**. Let $S = \{A_1, \ldots, A_k\} \subset V$ where $V$ is a vector space. The **span of $S$**, $\text{Span}(S) = \{\sum_{j=1}^{k} \alpha_j A_j : \alpha_j \in \mathbb{R} \text{ for } j = 1, \ldots, k\}$, is the set of all linear combinations of the elements of $S$.

3. **The Row Space of a Matrix $A$**. Let $A \in M(m,n)$ denote an $(m \times n)$-matrix. Say $A = \begin{bmatrix} A^1 \\ A^2 \\ \vdots \\ A^m \end{bmatrix}$. The row space of $A$ is the span of the set $\{A^1, \ldots, A^m\}$; i.e.
   \[ \text{Row}(A) = \text{Span}\{A^1, \ldots, A^m\} \].

4. **The Column Space of a Matrix $A$**. Let $A \in M(m,n)$ denote an $(m \times n)$-matrix. Say $A = [A_1, A_2, \ldots, A_n]$. The **column space of $A$** is the span of the set $\{A_1, \ldots, A_n\}$; i.e.
   \[ \text{Col}(A) = \text{Span}\{A_1, \ldots, A_n\} \].

5. **The Null Space of a Matrix $A$**. Let $A \in M(m,n)$ denote an $(m \times n)$-matrix. Say $A = [A_1, A_2, \ldots, A_n]$. The **null space of $A$** is the set
   \[ \text{Null}(A) = \{X \in M(n,1) : AX = 0\} \].
6. **A Linearly Independent Set of Vectors.** The set \( S = \{A_1, A_2, \ldots, A_k\} \) is said to be *linearly independent* if whenever

\[
\alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_k A_k = 0
\]

it follows that

\[
\alpha_1 = \alpha_2 = \cdots = \alpha_k = 0.
\]

**Computational questions**

1. Typical first problem:

   The reduced row echelon form of the matrix \( A \) that is associated to the homogeneous system of equations

   \[
   \begin{align*}
   5x + 4y - 2z &= 0 \\
   x + 2y + 2w &= 0
   \end{align*}
   \]

   is

   \[
   \begin{bmatrix}
   1 & 0 & -\frac{2}{3} & -\frac{4}{3} \\
   0 & 1 & \frac{1}{3} & \frac{5}{3}
   \end{bmatrix}
   \]

   (a) Determine the solution set.
   (b) Give the spanning set for the column space.

2. Solve the system of equations

   \[
   \begin{align*}
   x + y + z &= 1 \\
   2x + y + z &= 3 \\
   5x + 3y + 3z &= 6
   \end{align*}
   \]

   Determine the column space of the coefficient matrix.

3. Make sure that you can work all of the problems on Homework 4. For the small examples (two equations in two unknowns) you should be able to row reduce by multiplying by the corresponding operations as applied to the \( (2 \times 2) \) identity matrix.

4. Make sure that you can perform the matrix products and similar computations from Homework 3.

5. Understand the answers to the TF questions on page 14.

6. Make sure that you can do P. 16 # 13,14,23; P. 36 # 7,8; P. 58 # 1, #3 (f) use matrix multiplication; p. 80 # 1, 5, 10, 14.