

Math 125 Carter Test 1 Fall 2005

General Instructions: Write your name on only the outside of your blue book. Put your test paper inside your blue book as you leave. Solve each of the following problems point values (out of 100 points) are indicated on the problems.

1. Use the rules for computing the limit for each of the following problems (*10 points each*).

(a)

$$\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 1}$$

(b)

$$\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$$

(c)

$$\lim_{h \rightarrow 0} \frac{\sqrt{x + h} - \sqrt{x}}{h}$$

2. (*10 points*) Tell me what it means to say that a function, $y = f(x)$ approaches a limit, L as x approaches a .
3. (*10 points*) Given that the net investment portfolio of a 35 year old executive is worth \$200,000, and her expected rate of return is 7% continuously compounded interest per year, how long will it be until her portfolio exceeds \$1,000,000? How much will she have at retirement at age 70?
4. (*10 points*) Determine the domain of the function $y = \frac{x^2 - 4}{x^2 + 4x + 3}$. Determine all of its asymptotic behavior, and sketch the graph.
5. (*10 points*) Give a proof that

$$\lim_{h \rightarrow 0^+} \frac{\sin(h)}{h} = 1.$$

6. (*10 points each*) Determine the behavior of the function at $x = 1$ for each of the following. Is the function continuous at that point. If so, explain? If not, then why?

(a)

$$f(x) = \begin{cases} x^3 + 2 & \text{if } x \leq 1 \\ 3x & \text{if } 1 < x \end{cases}$$

(b)

$$f(x) = \begin{cases} \sin(x) & \text{if } x \leq 1 \\ 24x + 8 & \text{if } 1 < x \end{cases}$$

7. Solve the equation (*5 points each*)

(a)

$$\ln(x) + \ln(x + 1) = -3$$

(b)

$$e^x = 2 \times 10^{10}$$