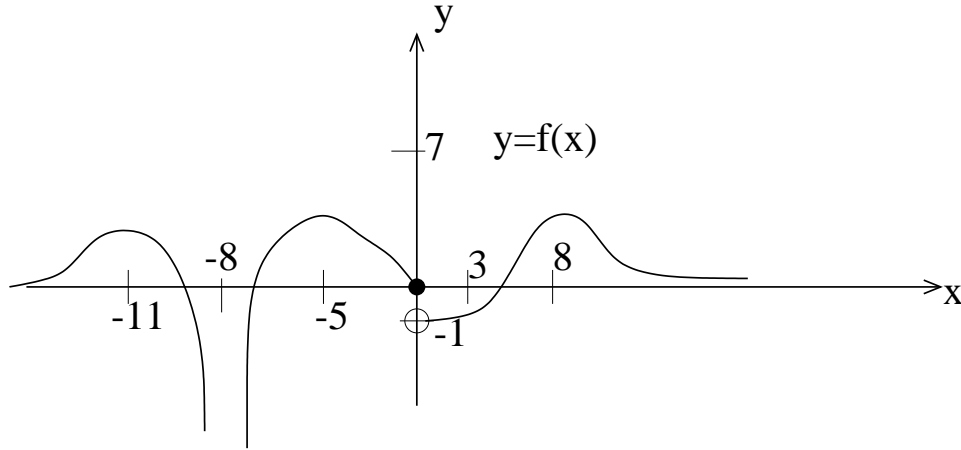


Math 125 Carter Test 1 Spring 2003

1. (16 points total) For the graph of the function illustrated in below determine the following:



- (a) $\lim_{x \rightarrow -8^-} f(x)$
 (b) $\lim_{x \rightarrow -8^+} f(x)$
 (c) $\lim_{x \rightarrow 0^-} f(x)$
 (d) $\lim_{x \rightarrow 0^+} f(x)$
 (e) $\lim_{x \rightarrow -\infty} f(x)$
 (f) $\lim_{x \rightarrow \infty} f(x)$
 (g) $f(0)$
 (h) At which points is $y = f(x)$ discontinuous?
 (i) List the x -coordinates of all the critical points.
2. Compute the following limits. Give justifications for your answers. (4 points each)

- (a) $\lim_{x \rightarrow \infty} \frac{2x^3}{5x^4 + 2x^3 - 2}$
 (b) $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3}$
 (c) $\lim_{x \rightarrow 0} \frac{\sqrt{x+1} - 1}{x}$
 (d) $\lim_{x \rightarrow 1} 3x^2 - 2x + 4$

3. (8 points) Prove by induction,

$$1 + 3 + \cdots + (2N - 1) = N^2.$$

4. (10 points) For a function $y = f(x)$ give the algebraic expression for the Newton quotient at the point $x = a$.
5. (10 points) Compute the derivative $f'(x)$ by computing the limit of the Newton quotient:

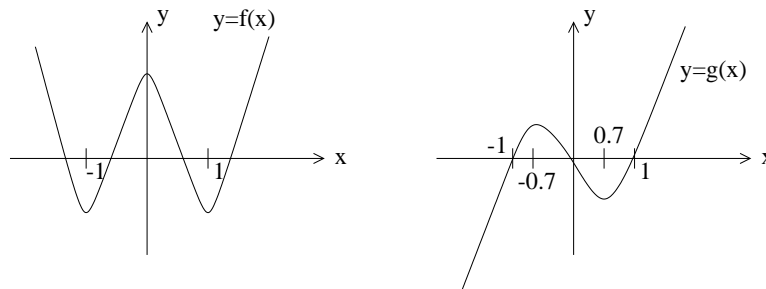
$$f(x) = \frac{1}{x+2}$$

6. (10 points total) The following expressions are the derivatives of a function $y = g(x)$ at a point $x = b$ for various functions. In each case identify $g(x)$ and the point b . DO NOT COMPUTE THE LIMITS.

(a) $\lim_{h \rightarrow 0} \frac{\ln(2+h) - \ln(2)}{h}$

(b) $\lim_{h \rightarrow 0} \frac{\cos(\frac{\pi}{3} + h) - 1/2}{h}$

7. (30 points total) Consider the graphs of $y = f(x)$ and $y = g(x)$ that are depicted on the left and right below, respectively. Answer the questions that follow with complete sentences or grammatically correct notation.



- (a) ¿How many critical points does $y = f(x)$ have?
- (b) ¿How many critical points does $y = g(x)$ have?
- (c) ¿On what intervals is $y = f(x)$ increasing? Decreasing?
- (d) ¿On what intervals is $y = g(x)$ increasing? Decreasing?
- (e) ¿For which values of x is the derivative, $f'(x)$ positive? Negative?
- (f) ¿For which values of x is the derivative, $y = g(x)$ positive? Negative?
- (g) ¿For which values of x is it true that $0 < g(x)$?
- (h) ¿For which values of x is it true that $0 < f(x)$?
- (i) True or False: ¿ $f(x) = g'(x)$?