**Math 227  Carter  Test 2  Fall 2011**

**General instructions.** Do all your work in your blue books. Write your solutions in your blue book. Show all work. Write your name on only the outside of your blue book. Please insert this sheet into your blue book as you leave. Write neatly, and use complete sentences when appropriate. My hope is that you do well on this exam.

For variety, you might try some tomato salsa on your eggs in the morning.

1. *(15 points)* Compute the equation to the line that is tangent to the helix

   \[ \vec{r}(t) = (4 \cos (t), t, 4 \sin (t)) \]

   at time \( t = \frac{\pi}{3} \). (Hint: use the variable \( s \) to parametrize the line).

2. *(15 points)* A water polo ball at the surface of the water is thrown at an angle of 45° from the surface of the water to a point just below the top cross-bar of the goal which is 1.5 meters from the water’s surface. The goal 15 meters away. At what initial velocity should the ball be thrown? Take the acceleration due to gravity to be \(-10 \) meters per square second.

3. Consider the function \( f(x, y) = x^2 - 6x - y^2 + 8y \).
   
   (a) *(5 points)* Letting \( y \) be the independent variable (horizontal axis), sketch the intersection of the surface with the vertical plane \( x = 0 \).
   
   (b) *(5 points)* Letting \( x \) be the independent variable (horizontal axis), sketch the intersection of the surface with the vertical plane \( y = 0 \).
   
   (c) *(5 points)* Sketch the intersection of the surface with the plane \( z = -8 \).
   
   (d) *(5 points)* Sketch the intersection of the surface with the plane \( z = 6 \).
   
   (e) *(5 points)* Determine the critical point(s) of \( f(x, y) \).
   
   (f) *(5 points)* Use the 2nd derivative test to determine if the critical point(s) yield local maxima, local minima, or saddle points.
   
   (g) *(5 points)* Compute the derivative for the composition of \( z = f(x, y) \) with the plane curve \( \vec{r}(t) = (2 \cos (t) + 3, 2 \sin (t) + 4) \), for \( t \in [0, 2\pi] \).
   
   (h) *(5 points)* Compute the maximal and minimal values for the composition of \( z = f(x, y) \) restricted to the plane curve \( \vec{r}(t) = (2 \cos (t) + 3, 2 \sin (t) + 4) \), for \( t \in [0, 2\pi] \).

4. *(15 points)* Compute the maximum and minimum values for the function \( f(x, y, z) = 3x + 2y + 5z \) over the sphere \( x^2 + y^2 + z^2 = 1 \). Leave your answer in terms of radicals.

5. *(15 points)* Let

   \[ \rho(x, y, z) = d(1 - \frac{x}{a} - \frac{y}{b} - \frac{z}{c}) \]

   denote the density (measured in grams per cubic centimeters) of a point in space that is bounded by the planes \( z = 0, y = 0, x = 0 \), and above by the plane \( z = c(1 - \frac{x}{a} - \frac{y}{b}) \).

   Write down a triple integral that can be used to compute the mass of the region.

6. *(20 points, extra credit)* Compute the integral for the previous problem.