1. Define the following terms (5 points each):
   (a) The Row Space of a Matrix $A$.
   (b) The Column Space of a Matrix $A$.
   (c) The Null Space of a Matrix $A$.

2. State the rank/nullity theorem (5 points).

3. (10 points). Express the matrix
   \[
   M = \begin{bmatrix} 4 & 7 \\ 7 & 9 \end{bmatrix}
   \]
   as a linear combination of the matrices $A, B,$ and $C$ where
   \[
   A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 \\ 4 & 5 \end{bmatrix}.
   \]

4. Let
   \[
   A = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 2 & 4 & 3 & 7 & 7 & 4 \\ 1 & 2 & 2 & 5 & 5 & 6 \\ 3 & 6 & 6 & 15 & 14 & 15 \end{bmatrix}.
   \]
   And let $M_5$ denote the $5 \times 5$ matrix that consists of the first 4 columns of $A$. Given that a row echelon form of $A$ is
   \[
   A' = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 & 1 \\ 0 & 0 & 1 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
   \]
   (a) (10 points) Find a basis for the column space of $M_5$.
   (b) (10 points) Write the remaining columns of $M_5$ as linear combinations of the basis that you found.
   (c) (10 points) determine the kernel (null space) of $A$. 


5. (10 points) Find the dimension and a basis for the solution space $W$ for the homogeneous system of equations:

\[
\begin{align*}
x + y + 2z &= 0 \\
2x + 3y + 3z &= 0 \\
x + 3y + 5z &= 0
\end{align*}
\]

6. (10 points) Let $\mathbb{R}^3 \xrightarrow{F} \mathbb{R}^4$ denote the linear mapping that is given by

\[
F \begin{bmatrix} \ x \\ \ y \\ \ z \\ \ t \end{bmatrix} = \begin{bmatrix} x - y + z + t \\ x + 2z - t \\ x + y + 3z - 3t \end{bmatrix}.
\]

Find a basis for the image of $F$, and find a basis for the null space (kernel) of $F$.

7. (10 points) Suppose that the matrix

\[
A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}
\]

represents a linear operator $\mathbb{R}^2 \xleftarrow{F} \mathbb{R}^2$ relative to the usual basis

\[
\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}
\]

for $\mathbb{R}^2$. Find the matrix $B$ that represents $A$ with respect to the basis

\[
\mathcal{S} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}.
\]

8. (10 points) Determine the equation of the plane whose solution space is the span of the set of vectors $\{[0, 1, -1]^t, [-1, 1, 0]^t\}$.

9. (10 points) Determine bases for the column space, row space, and null space of the matrix

\[
A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & -4 & 4 \\ 1 & 3 & 0 & 4 \end{bmatrix}.
\]