Math 227    Carter    Test 3

Do all your work in your blue book. Write your name on only the outside of your blue book. Do not write on this sheet, but insert it into the blue book as you leave. Write neat complete solutions to all problems. Be careful! If you are cutting chili peppers, wash your hands before and after going to the bathroom.

1. Consider the quadratic surface \( f(x, y) = -(x + 8)^2 - (y - 15)^2 \).
   (a) Sketch the \( z = 0 \), \( z = -1 \), and \( z = -17 \) levels of the surface (5 points).
   (b) Compute the gradient \( \nabla f \) (5 points).
   (c) Find the critical point(s) of \( f(x, y) \) (5 points).
   (d) Compute the determinant of the Hessian \( H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix} \) at the critical point (5 points).
   (e) Is the critical point a local maximum, minimum, or neither? Explain why it (5 points).
   (f) Sketch the gradient vector field (without regard to magnitude), along the \( z = 17 \) level (5 points).
   (g) Compute the work done in moving a particle once around an ellipse \( \frac{x^2}{9} + \frac{y^2}{16} = 1 \) in the gradient field (5 points).

2. Calculate the volume that is enclosed by the sphere \( x^2 + y^2 + z^2 = 25 \) and the cylinder \( r = 5 \cos \theta \) (10 points).

3. Set up an integral that computes the surface area of the region of a sphere \( x^2 + y^2 + z^2 = 16 \) that lies above the square \( S = \{ (x, y) : -1 \leq x \leq 1 \quad \& \quad -1 \leq y \leq 1 \} \) (15 points).

4. Recall, that the differential form \( ds = \sqrt{(dx)^2 + (dy)^2} \) dt. Compute the line integral

\[ \int_C x \, y^4 \, ds \]

where \( C \) is the right half of the circle \( x^2 + y^2 = 16 \). When I computed this, I got \( \frac{8192}{5} \). Is my answer correct (20 points)?

5. (20 points) Determine whether or not \( \vec{F} \) is conservative. If so, find a potential function.

\( \vec{F}(x, y) = (3 + 2xy) \hat{i} + (x^2 - 3y^2) \hat{j} \)