Find the value or values $c$ that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the statement of the Mean Value Theorem for the functions and intervals.

1. 

$$f(x) = x + \frac{1}{x}, \quad [1/2, 1].$$

Solution.

$$f(1) = 1 + 1/1 = 2.$$

$$f(1/2) = 1/2 + \frac{1}{\frac{1}{2}} = 1/2 + 2 = 5/2.$$

$$f(1) - f(1/2) = 2 - 5/2 = -1/2.$$

$$\frac{1 - 1/2}{1 - 1/2} = \frac{-1/2}{1/2} = -1$$

Meanwhile

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}.$$ 

Solve

$$\frac{x^2 - 1}{x^2} = -1.$$

$$x^2 - 1 = -x^2.$$

$$2x^2 - 1 = 0.$$

$$x = \pm 1/\sqrt{2}.$$

But we only want the positive value,

$$x = \frac{1}{\sqrt{2}}$$
2. \[ f(x) = \sqrt{x-1}, \quad [1, 3]. \]

Solution.

\[ f(3) = \sqrt{2}. \]

\[ f(1) = 0. \]

\[ \frac{f(3) - f(1)}{3 - 1} = \frac{\sqrt{2}}{2}. \]

Meanwhile,

\[ f'(x) = \frac{1}{2\sqrt{x-1}}. \]

Solve

\[ \frac{1}{2\sqrt{x-1}} = \frac{\sqrt{2}}{2}. \]

\[ \frac{1}{\sqrt{x-1}} = \sqrt{2}. \]

\[ \frac{1}{x - 1} = 2. \]

\[ 1 = 2x - 2 \]

\[ 3/2 = x. \]