

Abstract

Matrices over Finite Fields XV: Generators and Relations

We present the general linear group of 2 times 2 matrices over a finite field via generators and relations. In doing that, we take the viewpoint that the Borel subgroup is something well understood, as it is the semidirect product of the additive group of the field with the Cartan subgroup, which in turn is the direct product of two copies of the multiplicative group of the field. To get the full group from the Borel subgroup, it is sufficient to adjoin a single new generator. This element is compatible with the Borel subgroup via various compatibility conditions, which in turn give rise to the necessary relations.