

Practice problems for Calculus I Test I.

1. Write the precise  $\epsilon$ - $\delta$  definition of the following:  $\lim_{g \rightarrow f} x(g) = a$ .
2. Illustrate the definition of  $\lim_{x \rightarrow 4} (3x - 2) = 10$  by finding a value of  $\delta$  that corresponds to  $\epsilon = 1/2$ . Justify your choice.
3. Use the definition of limit to prove that  $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$ .
4. Sketch the graphs (on a single  $xy$ -plane) of the functions  $y = x^{1/3}$ ,  $y = x^{1/4}$  and  $y = x^{1/5}$ , indicating which curve corresponds to which function.
5. For the functions  $f(x) = \frac{1}{x}$  and  $g(x) = x^3 + 3x$ , calculate the compositions  $f \circ g$  and  $g \circ f$  and find the domain of each.
6. Calculate the limit  $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$ .
7. Does  $\lim_{x \rightarrow 0} (x \sin(1/x))$  exist? Why or why not?
8. State the intermediate value theorem.
9. Show that the equation  $x^5 - x^2 - 4 = 0$  has at least one solution.
10. Let  $f(x)$  be the function in problem #3 on page 46 (the blue one). Sketch the graphs of  $y = 2f(x)$ ,  $y = f(x + 3)$ , and  $y = -f(x) + 1$ .
11. Let  $f(x)$  be the function whose graph is on page 122 (#1 in the chapter 2 review exercises).
  - a) Find all  $x$  for which the function is discontinuous at  $x$ .
  - b) Find all  $a$  for which  $\lim_{x \rightarrow a^+} f(x)$  does not exist.
  - c) Find all  $a$  for which  $\lim_{x \rightarrow a^-} f(x)$  does not exist.
  - d) Find all  $a$  for which  $\lim_{x \rightarrow a} f(x)$  does not exist.
12. Find  $\lim_{x \rightarrow 0} \cos(x^2 - 2x)$ , justifying your steps.
13. What, if anything, is wrong with the following:

$$\lim_{x \rightarrow 0} \left( \frac{x}{\sin x} \right) = \frac{\lim_{x \rightarrow 0} (x)}{\lim_{x \rightarrow 0} (\sin x)} = \frac{0}{\lim_{x \rightarrow 0} (\sin x)} = 0.$$

14. Find the slope of the line tangent to the graph of  $y = x^2 - 3x$  where  $x = 4$ . Use both formulas.