

Introduction to Linear Independence

A collection of vectors $S = \{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly independent if there is no way to write the zero vector $\mathbf{0}$ as a non-trivial linear combination of the vectors in S . In other words, S is linearly independent if the equation $a_1\mathbf{v}_1 + \dots + a_k\mathbf{v}_k = \mathbf{0}$ has only one solution, namely $a_1 = \dots = a_k = 0$.

How-to

To determine if a set of vectors $S = \{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly independent, one sets up the equation $\mathbf{v} = a_1\mathbf{v}_1 + \dots + a_k\mathbf{v}_k$ and tries to solve for a_1, \dots, a_k . Note that you will always find one solution, namely $a_1 = \dots = a_k = 0$. If you find any other solutions (and thus infinitely many solutions) then the set is linearly dependent; otherwise it is linearly independent.

Note: How you actually determine if there are such a_1, \dots, a_k depends on the context; i.e., it depends on the vector space V you are actually dealing with. In the examples we will do, it will always come down to solving a system of linear equations.

For example, if $V = \mathbf{R}^n$, then the equation $a_1\mathbf{v}_1 + \dots + a_k\mathbf{v}_k = \mathbf{0}$ has an $n \times 1$ vector on either side, and so turns into a system of n linear equations obtained by setting each of the entries of the left side equal to zero.

If $V = P_n$, then the equation $a_1\mathbf{v}_1 + \dots + a_k\mathbf{v}_k = \mathbf{0}$ has an n degree polynomial on either side, and so turns into a system of $n + 1$ linear equation obtained by setting all of the coefficients of the polynomial on the left equal to zero.