Practice problems for Linear Algebra Test I.

1. Let $A = \begin{bmatrix} -1 & 0 \\ 3 & -5 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 5 & -4 & -3 & 3 \end{bmatrix}$.

a) Find A^T b) Find AB c) Find $7A - 2B$

ANSWER:

$$A^T = \begin{bmatrix} -1 & 3 & 6 \\ 0 & -5 & 7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & -1 & -2 & -2 \\ -25 & 23 & 21 & -9 \\ 35 & -22 & -9 & 33 \end{bmatrix}$$

$7A - 2B$ is not defined becasue A and B are not the same size.

2. a) Find the augmented matrix corresponding to the following system of linear equations:

$$-4x_1 + 3x_2 + 5x_3 - 6x_4 = 0$$

$$x_1 - 3x_3 + 5x_4 = 2$$

$$-x_1 + 3x_2 - 4x_3 + 9x_4 = 7$$

ANSWER:

$$\begin{bmatrix} -4 & 3 & 5 & -6 & 0 \\ 1 & 0 & -3 & 5 & 2 \\ -1 & 3 & -4 & 9 & 7 \end{bmatrix}$$

b) Use Gaussian elimination to put the above matrix in row echelon form (show all row reductions).

ANSWER:

One answer is

$$\begin{bmatrix} 1 & 0 & -3 & 5 & 2 \\ 0 & 1 & -7/3 & 14/3 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c) Use Gauss-Jordan elimination to put the above matrix in reduced row echelon form (show all row reductions).

ANSWER:

$$\begin{bmatrix} 1 & 0 & -3 & 5 & 0 \\ 0 & 1 & -7/3 & 14/3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Use row reductions to find the inverses of the following matrix: $\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & -5 \\ 2 & 0 & 1 \end{bmatrix}$

ANSWER:

$$\begin{bmatrix} -1 & 0 & 1 \\ -12 & -1 & 7 \\ 2 & 0 & -1 \end{bmatrix}$$

4. Determine for which values of a the following system has 0 solutions, 1 solution, and infinitely many solutions.

$$5x + 5y = 10$$

$$4x + a^2 y = 4 + 2a$$

ANSWER:

When $a = 2$ there are infinitely many solutions. When $a = -2$ there are no solutions. Otherwise there is a unique solution.

5. Find all 2×2 matrices $A = \begin{bmatrix} 0 & b \\ c & d \end{bmatrix}$ with the property that $A^2 = I_2$.

ANSWER:

$$\begin{bmatrix} 0 & b \\ 1/b & 0 \end{bmatrix}$$

6. a) Suppose $V = \mathbb{R}^2$ (the set of 2×1 matrices) with $\oplus$ defined as ordinary matrix addition, but with $\odot$ defined as $c \odot \mathbf{v} = \mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Is V a real vector space with these operations? Explain.

ANSWER:

No, because property (8) is not satisfied (because for any vector $\mathbf{u} \neq \mathbf{0}$ we have $1 \cdot \mathbf{u} = \mathbf{0} \neq \mathbf{u}$).

b) Verify directly that M_{22} (the set of 2×2 real matrices) is a real vector space (under ordinary matrix addition and scalar multiplication).

c) Show that matrices of the form $\begin{bmatrix} 0 & b \\ c & 0 \end{bmatrix}$ form a subspace of the vector space M_{22} (under ordinary matrix addition and scalar multiplication).