Answers to homework 20 problems

14.1 #19-24: 19-VI, 20-II, 21-IV, 23-V, 24-III

14.2 #18: We have that $\mathbf{r}'(t) = \langle 2/\sqrt{t}, 2t, 1\rangle$, so $\mathbf{r}'(1) = \langle 2, 2, 1\rangle$, whose length is $\sqrt{9} = 3$. Thus we have

$$\mathbf{T}(1) = \langle 2/3, 2/3, 1/3\rangle.$$

14.2 #24: The curve is $\mathbf{r}(t) = \langle t^2 - 1, t^2 + 1, t + 1\rangle$, so $\mathbf{r}'(t) = \langle 2t, 2t, 1\rangle$. The point $(-1, 1, 1)$ corresponds to $t = 0$, so we have $\mathbf{r}'(0) = \langle 0, 0, 1\rangle$. The tangent line goes through $(-1, 1, 1)$ in the direction of $\langle 0, 0, 1\rangle$, so the line is given by $\langle -1, 1, 1 + t\rangle$.

14.2 #40: Integrating we have $\mathbf{r}(t) = \langle -\cos t, -\sin t, t^2\rangle + \mathbf{C}$. Using the fact that $\langle 1, 1, 2\rangle = \mathbf{r}(0) = \langle -1, 0, 0\rangle + \mathbf{C}$, we find that $\mathbf{C} = \langle 2, 1, 2\rangle$. Thus

$$\mathbf{r}(t) = \langle 2 - \cos t, 1 - \sin t, 2 + t^2\rangle.$$