Answers to homework 21 problems

14.3 #2: $\mathbf{r}'(t) = \langle 2t, t\sin t, t\cos t\rangle$, which has length $\sqrt{5}$. Thus we have

$$L = \int_0^\pi |\mathbf{r}'(t)|\,dt = \int_0^\pi \sqrt{5}\,t\,dt = \frac{\sqrt{5}\,\pi^2}{2}.$$

14.3 #14: (a) $|\mathbf{r}'(t)| = |\langle 2t, t\sin t, t\cos t\rangle| = \sqrt{5}\,t$. Thus

$$\mathbf{T}(t) = \frac{1}{\sqrt{5}}\langle 2, \sin t, \cos t\rangle.$$

Then $|\mathbf{T}'(t)| = \frac{1}{\sqrt{5}}|\langle 0, \cos t, -\sin t\rangle| = \frac{1}{\sqrt{5}}$. Thus

$$\mathbf{N}(t) = \langle 0, \cos t, -\sin t\rangle.$$

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{1}{5t}$.

14.3 #40: The point $(1, 0, 1)$ corresponds to $t = 0$, and $\mathbf{r}(t) = e^t\langle 1, \sin t, \cos t\rangle$. Careful calculation gives

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{3}}\langle 1, \sin t + \cos t, \cos t - \sin t\rangle.$$

At $t = 0$ we have

$$\mathbf{T}(0) = \left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle.$$

Similarly, we find that

$$\mathbf{N}(t) = \frac{1}{\sqrt{2}}\langle 0, \cos t - \sin t, -\sin t - \cos t\rangle,$$

so that

$$\mathbf{N}(0) = \left\langle 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right\rangle.$$

Crossing these gives

$$\mathbf{B}(0) = \left\langle -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle.$$

14.3 #41: The point $(0, \pi, -2)$ corresponds to $t = \pi$. Using $\mathbf{T}(\pi)$ as a normal vector, we find that the normal plane is given by

$$y - 6x = \pi.$$

Using $\mathbf{B}(\pi)$ as a normal vector for the osculating plane, we obtain

$$x + 6y = 6\pi.$$