Answers to homework 22 problems

14.4 #12: $\mathbf{v}(t) = \langle 2t, 1/t, 1\rangle$, $\mathbf{a}(t) = \langle 2, -1/t^2, 0\rangle$, and $|\mathbf{v}(t)| = \sqrt{4t^2 + t^{-2} + 1}$.

14.4 #16: Integrating we obtain $\mathbf{v}(t) = \langle 0, 0, -10t\rangle + \mathbf{C}_1$. Using the fact that $\langle 1, 1, -1\rangle = \mathbf{v}(0) = \langle 0, 0, 0\rangle + \mathbf{C}_1$, we find that

$$\mathbf{v}(t) = \langle 1, 1, -1 - 10t\rangle.$$

Similarly, one finds that

$$\mathbf{r}(t) = \langle t + 2, t + 3, -5t^2 - t\rangle.$$

14.4 #28: We put the origin at homeplate and the path of the ball in the xy-plane. Then we have $\mathbf{a}(t) = \langle 0, -32\rangle$, so $\mathbf{v}(t) = \langle 0, -32t\rangle + \mathbf{C}_1$. Using trig, we find that $\mathbf{v}(0) = \langle 115\cos 50°, 115\sin 50°\rangle \approx \langle 73.92, 88.1\rangle$. Thus

$$\mathbf{v}(t) = \langle 73.92, 88.1 - 32t\rangle.$$

Thus $\mathbf{r}(t) = \langle 73.92t, 88.1t - 16t^2\rangle + \mathbf{C}_2$. Using the fact that $\mathbf{r}(0) = \langle 0, 3\rangle$, we find that

$$\mathbf{r}(t) = \langle 73.92t, 3 + 88.1t - 16t^2\rangle.$$

The ball reaches the fence when the x-coordinate is 400, so when $t \approx 5.41$. The height at this time is approxmately 11.2, which, being greater than ten, means that the ball clears the fence.

14.4 #34:

$$a_T = \frac{4t}{\sqrt{4t^2 + 10}} \qquad a_N = \frac{2\sqrt{10}}{\sqrt{4t^2 + 10}}$$