Answers to homework 22 problems

15.7 #4: We have that $f_x = 3 - 3x^2$ and $f_y = -4y + 4y^3$. For both of these to be zero, we must have $x = \pm 1$ and $y = 0, \pm 1$. We therefore have six critical points

$$(1,0), (-1,0), (1,1), (-1,1), (1,-1), (-1,-1).$$

Note that $f_{xx} = -6x$, $f_{yy} = 12y^2 - 4$, and $f_{xy} = 0$, so that the values of D for these points are, in order,

$$D = 24, -24, -48, 48, -48, 48.$$

It follows, by checking the sign of f_{xx} at those points where $D > 0$, that f has a local maximum at $(1,0)$, local minima at $(-1,1)$ and $(-1,-1)$, and saddles at $(1,1)$, $(1,-1)$, and $(-1,0)$. I leeave it to you to see identify these phenomena in the contour diagram.

15.7 #6: We have that $f_x = 3x^2y + 24x$ and $f_y = x^3 - 8$, so that the only critical point is $(2,-4)$. Because $f_{xx} = 6xy + 24$, $f_{yy} = 0$, and $f_{xy} = 3x^2$, we have that $D = -144 < 0$, so this point is a saddle.

15.7 #28: The only critical point inside D is $(2,1)$, where $f(2,1) = 1$. Along the line $x = 1$, we have that $f = 2 - y$, so that the maximum value is $f(1,0) = 2$ and the minimum value is $f(1,4) = -2$. Along the line $y = 0$ we have that $f = 3 - x$, so that the maximum value here is $f(1,0) = 2$ and the minimum value is $f(5,0) = -2$. Along the line $y = 5 - x$, we have that $f = -x^2 + 6x - 7$, whch, on the interval where $1 \le x \le 5$, has a maximum value $f(3,2) = 2$ and a minimum of $f(1,4) = f(5,0) = -2$. It follows that the absolute maximum of f on D is $f(1,0) = f(3,2) = 2$, and the absolute minimum of f on D is $f(1,4) = f(5,0) = -2$.

15.7 #46: Surface area is $A = 2(xy + xz + yz) = 64$, or $z = \dfrac{32 - xy}{x + y}$. Thus the volume is

$$V = xyz = xy\frac{32 - xy}{x + y}.$$

The critical value of this function (which intuitively must be a maximum) occirs when $x = y = z = \frac{8}{\sqrt{6}}$.