

## LINE INTEGRALS

1. Integrating a function along a curve:

$$\int_C f \, ds = \int_a^b f(\mathbf{r}(t)) |\mathbf{r}'(t)| \, dt.$$

2. Integrating a vector field along a curve:

- (a) In general, evaluate as

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t)) \cdot \mathbf{r}'(t) \, dt.$$

- (b) If  $C$  is closed, try Green's/Stokes' theorem. Find a surface  $S$  with boundary  $C$  and use

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot d\mathbf{S}.$$

If you're in the plane and want the normal component, you may use the divergence form of Green's theorem

$$\int_C (\mathbf{F} \cdot \mathbf{n}) \, ds = \iint_D \text{div } \mathbf{F} \, dA.$$

- (c) If  $\mathbf{F}$  is conservative, use the fundamental theorem for line integrals. Either replace  $C$  with any other path having the same endpoints and evaluate directly, as in (a), or find a potential function  $f$  and use

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

- (d) If  $\mathbf{F}$  is conservative *and*  $C$  is closed, the answer is zero, by the fundamental theorem for line integrals.

## SURFACE INTEGRALS

1. Integrating a function over a surface:

$$\iint_S f \, dS = \iint_D f(\mathbf{r}(u, v)) |\mathbf{r}_u \times \mathbf{r}_v| \, dA.$$

2. Integrating a vector field over a surface:

- (a) In general, evaluate as

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D \mathbf{F}(\mathbf{r}(u, v)) \cdot (\mathbf{r}_u \times \mathbf{r}_v) \, dA.$$

- (b) If  $S$  is closed, try the divergence theorem. Find a solid  $E$  with boundary  $S$  and use

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div } \mathbf{F} \, dV.$$

- (c) If  $\mathbf{F}$  is a curl, use Stokes' theorem. Either replace  $S$  with any other surface having the same boundary curve  $C$  and evaluate directly, as in (a), or find a vector field  $\mathbf{G}$  so that  $\mathbf{F} = \text{curl } \mathbf{G}$  and use

$$\iint_S (\text{curl } \mathbf{G}) \cdot d\mathbf{S} = \int_C \mathbf{G} \cdot d\mathbf{r}.$$

- (d) If  $\mathbf{F}$  is a curl *and*  $S$  is closed, the answer is zero, by Stokes' theorem.