Test 1 Solutions

Calculus II **September 7, 2007**

1. $\displaystyle\int_1^2 \frac{dx}{(3-5x)^2} =$

We use the substitution $u = 3-5x$, so that $du = -5\,dx$. Note that when $x=1$, we have $u=-2$, and when $x=2$, we have $u=-7$. Thus we have

$$\int_1^2 \frac{dx}{(3-5x)^2} = \int_{-2}^{-7} \frac{-1/5\,du}{u^2} = -\frac{1}{5}\int_{-2}^{-7} u^{-2}\,du = -\frac{1}{5}\left[-u^{-1}\right]_{u=-2}^{u=-7}$$

$$= \frac{1}{5}\left[\frac{1}{3-5x}\right]_{x=1}^{x=2}$$

$$= \frac{1}{5}\left(-\frac{1}{7}+\frac{1}{2}\right) = \frac{1}{14}.$$

2. $\displaystyle\int \cos^3(x)\,dx =$

We first use the identity $\sin^2(x)+\cos^2(x)=1$ to write

$$\int \cos^3(x)\,dx = \int (1-\sin^2(x))\cos(x)\,dx,$$

and then use the substitution $u=\sin(x)$, $du=\cos(x)\,dx$, obtaining

$$= \int (1-u^2)\,du = u - \frac{u^3}{3} + C = \sin(x) - \frac{1}{3}\sin^3(x) + C.$$

3. $\displaystyle\int \frac{dx}{x^2\sqrt{1-x^2}} =$

We use the trig substitution $x=\sin(u)$, $dx=\cos(u)\,du$, obtaining

$$\int \frac{dx}{x^2\sqrt{1-x^2}} = \int \frac{\cos(u)\,du}{\sin^2(u)\sqrt{1-\sin^2(u)}} = \int \frac{\cos(u)\,du}{\sin^2(u)\cos(u)}$$

$$= \int \frac{du}{\sin^2(u)} = \int \csc^2(u)\,du = -\cot(u) + C.$$

In order to express the answer in terms of x, we draw a right triangle. Because we have $\sin(u) = x$, we may assume the opposite side has length x and the hypotenuse has length 1. Thus the adjacent side has length $\sqrt{1-x^2}$. It follows that $\cot(u) = \dfrac{\sqrt{1-x^2}}{x}$, so the final answer is

$$-\frac{\sqrt{1-x^2}}{x} + C.$$