

Quiz 1 Solution

Calculus II

August 24, 2007

1. Write the following as an integral entirely in u , using the substitution $u = x - 3$. Then evaluate the integral.

$$\int_0^2 \frac{2x}{(x-3)^2} dx$$

From $u = x - 3$, we have also that $x = u + 3$, and $\frac{du}{dx} = 1$, so $du = dx$. When $x = 0$, we have that $u = -3$, and when $x = 2$, we have $u = -1$. Thus the integral becomes

$$\int_0^2 \frac{2x}{(x-3)^2} dx = \int_{-3}^{-1} \frac{2(u+3)}{u^2} du.$$

To evaluate this integral, we split the fraction into two fractions, and then we simplify using rules of exponents, obtaining

$$\begin{aligned} \int_{-3}^{-1} \left(\frac{2u}{u^2} + \frac{6}{u^2} \right) du &= \int_{-3}^{-1} \left(\frac{2}{u} + 6u^{-2} \right) du \\ &= \left[2 \ln |u| - 6u^{-1} \right]_{u=-3}^{u=-1} = (2 \ln(1) + 6) - (2 \ln(3) + 2) = \boxed{4 - 2 \ln(3)} \end{aligned}$$