

Quiz 2 Solution

Calculus II

August 31, 2007

1. Use a trigonometric identity and substitution to evaluate the following indefinite integral:

$$\int \tan^3(x) \sec^3(x) dx$$

We know we will either use $u = \tan(x)$ or $u = \sec(x)$. In the first case, we would have $du = \sec^2(x) dx$, so we would have to pull aside a $\sec^2(x) dx$ to be our du , and turn everything else into tangents. Unfortunately, this would leave only a single secant factor, which cannot conveniently be changed into tangents using the standard identity.

On the other hand, if we use the substitution $u = \sec(x)$, then we have that $du = \sec(x) \tan(x) dx$. Thus we pull aside a secant and a tangent to be our du , and turn the rest of the tangents into secants. Because there are three tangent factors, pulling one aside for du leaves two, which easily change to secants when the identity is applied. Thus we have

$$\begin{aligned} & \int \tan^3(x) \sec^3(x) dx \\ &= \int \tan^2(x) \sec^2(x) [\sec(x) \tan(x)] dx \\ &= \int (\sec^2(x) - 1) \sec^2(x) [\sec(x) \tan(x)] dx \\ &= \int (u^2 - 1) u^2 du \\ &= \int (u^4 - u^2) du \\ &= \frac{u^5}{5} - \frac{u^3}{3} + C \\ &= \frac{1}{5} \sec^5(x) - \frac{1}{3} \sec^3(x) + C. \end{aligned}$$