

Quiz 3 Solutions

Calculus II

September 14, 2007

1. Find the volume of the solid obtained by rotating the region between the curves $y = x$ and $y = x^2$ about the x -axis.

To use washers, slice the region vertically. Thus our variable is x , and the region runs from $x = 0$ to $x = 1$. The top curve in the region is $y = x$, and the distance from a point on this curve to the axis of rotation is exactly the y -coordinate of the point. Thus we have that the large radius of the washer is $R = x$. Similarly, the small radius is x^2 . The area of a typical washer is thus $\pi(x^2 - x^4)$. It follows that the total volume of this solid is

$$V = \int_0^1 (x^2 - x^4) dx = \frac{x^3}{3} - \frac{x^5}{5} \Big|_{x=0}^{x=1} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}.$$