

Quiz 4 Solutions

Calculus II

September 21, 2007

1. The tank shown on the board is full of water. Find the work required to pump the water out of the outlet. Use the fact that the cross-section of the tank that is x feet from the top of the tank is a rectangle with width $3 - x$ meters.

We will point the x -axis downward, with the origin at the top of the tank. Thus the depth of a typical slice is x . We are told that such a slice is a rectangle with width $3 - x$. The length is 8, so that the area of a slice at depth x is $8(3 - x) = 24 - 8x$. Then the volume of a thin slice is $(24 - 8x)\Delta x$, and its mass is $1000(24 - 8x)\Delta$, so the force required to move it is $(9.8)(1000)(24 - 8x)\Delta x$. The distance it needs to travel is exactly its depth, x , so that the work required to move this slice all the way to the top of the tank is $9800x(24 - 8x)\Delta x$. It follows that the total work required to empty the tank is

$$\begin{aligned} W &= 9800 \int_0^3 (24 - 8x)x \, dx = 9800 \int_0^3 (24x - 8x^2) \, dx \\ &= 9800 \left[12x^2 - \frac{8}{3}x^3 \right]_{x=0}^{x=3} = 9800(108 - 72) = 254,800. \end{aligned}$$