

Quiz 5

Name:

Calculus II

ID:

October 12, 2007

Instructions: Please be sure to write neatly, show your work, and put your answer in a box.

1. Evaluate the following indefinite integral using partial fractions:

$$\int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx$$

The denominator factors as $x^3 + 2x^2 = x^2(x + 2)$. It follows that we may write

$$\frac{5x^2 + 3x - 2}{x^2(x + 2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 2}.$$

To solve for the numerators, we multiply through by the denominator, obtaining

$$5x^2 + 3x - 2 = Ax(x + 2) + B(x + 2) + Cx^2.$$

Now we plug in the roots. When $x = 0$, this equation becomes

$$-2 = 2B \quad \Rightarrow \quad B = -1,$$

and when $x = -2$, it becomes

$$5(4) + 3(-2) - 2 = C(4) \quad \Rightarrow \quad C = 3.$$

Finally, to find A we plug in any random value of x . When $x = 1$, for instance, we have

$$5 + 3 - 2 = 3A + 3B + C = 3A + 3(-1) + 3 \quad \Rightarrow \quad A = 2.$$

Thus we have

$$\begin{aligned} \int \frac{5x^2 + 3x - 2}{x^3 + 2x^2} dx &= \int \frac{2}{x} dx + \int \frac{-1}{x^2} dx + \int \frac{3}{x + 2} dx \\ &= 2 \ln|x| + \frac{1}{x} + 3 \ln|x + 2| + C. \end{aligned}$$